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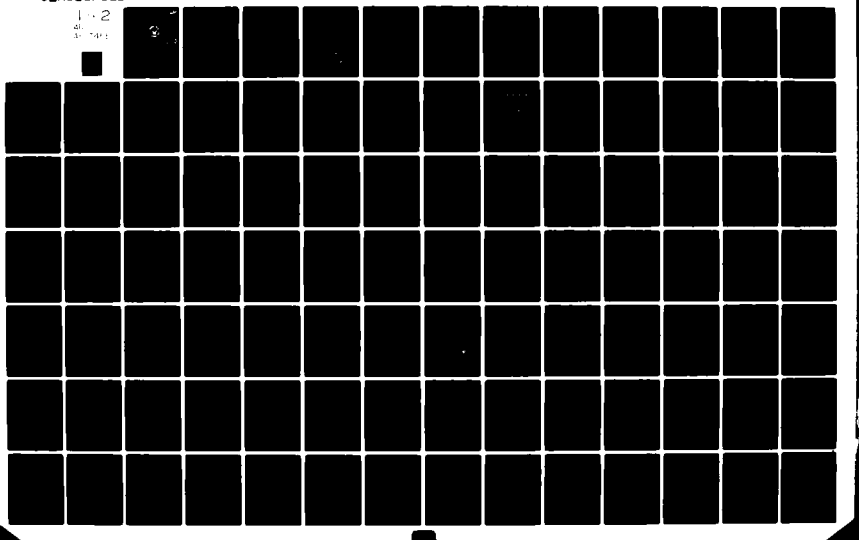

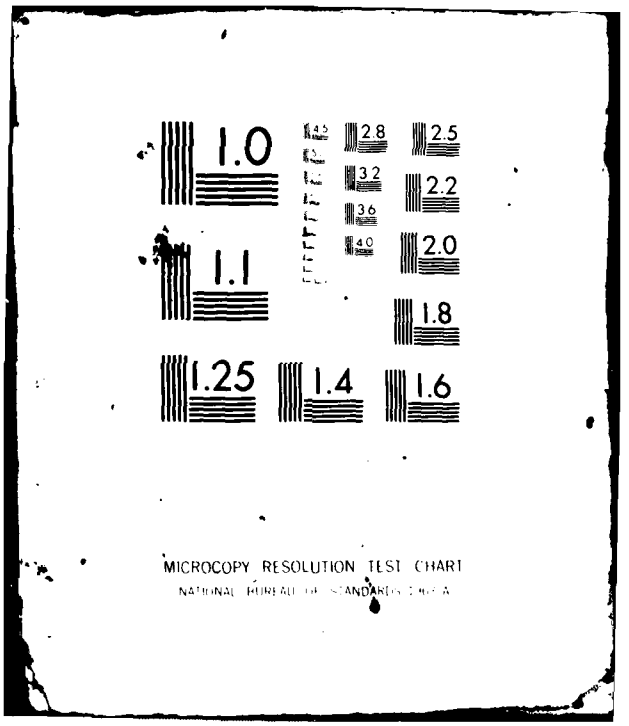
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THESIS

A COMPUTER ANALYSIS OF THE PLASMA-BOUNDARY LAYER BEHAVIOR OVER A POSITIVE ELECTRODE

by

Stephen Ted Van Brocklin

June 1981

Thesis Advisor: O. Biblarz

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Results clearly demonstrate the boundary layer nature of the species density at the electrode. A sheath length of 38 microns about a 35-volt active site and 5-20 microns along the non-active portion of the electrode is established. Joule heating is determined to be not important.

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A Computer Analysis of the Plasma-Boundary Layer Behavior Over a Positive Electrode

by

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Submitted in partial fulfillment of the requirements for the degree of

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ABSTRACT

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ABLE OF SYMBOLS

English Letters And Symbols

A	matrix of coefficients (Eq. C10)
c_{1-3}	non-dimensionalizing terms (Eqs.6,7 and 8)
D_s	diffusion coefficient of species-s
E	electric field
e	electron charge ($1.602\ 10^{-19}$ coulombs)
f	namely f(x)... a function of x (App. C)
J	current (amps/m^2)
$\mathbf{J}$	Jacobian (Eq. C5)
k	Boltzmann's constant ($1.38\ 10^{-23}$ V-coul/^oK)
L	characteristic diffusion length
ℓ_p	characteristic production length
n^*, N	stationary density state (App A, Sec C)
n_s	number density of species-s (m^{-3})
p_s	pressure of species-s ($p_s = n_s k T_s$)
R_e	Reynolds number
s	subscript denoting electron or ion species
T_s	temperature of species-s
u	velocity vector
v_d	drift velocity (Sec. III.A.1)
$\bar{v}_s$	average thermal velocity (App A, Sec C)
V_∞	free-stream velocity in the y-direction

Z	degree of ionization (Z=1 in this study)
z	column vector of solution set ϕ, n_e, n_i

Greek Characters and Symbols

α	thermal diffusivity (Sec. III.A.3)
α_i	recombination rate coefficient (App. B)
β	ratio of electron to the background gas temperature
δ	velocity boundary layer thickness (Sec. III.A.3)
ϵ_o	permitivity of free space (8.854×10^{-12} F/m)
λ_s	characteristic sheath length
μ_s	mobility of species-s
ν	dynamic viscosity (Sec. III.A.3)
ν_i	ionization rate coefficient (App. B)
ϕ	voltage potential (volts)
Φ	functional defined in App. C, to be minimized.
τ	characteristic time
τ_p	characteristic net production time
∞	refers to the free-stream condition
∇	mathematical gradient operator
Σ	mathematical summation
$\hat{}$	"caret" indicating a non-dimensional term
Δ	"delta" indicating a small increment or change

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I extend my gratitude to Professor O. Biblarz, who as my thesis advisor and friend, provided expert assistance, guidance and inspiration. With his direction and special counsel the objectives of this work were met and are compiled in these pages.

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I. INTRODUCTION

A. BACKGROUND

Losses at the electrode boundary regions play an important role in high energy density molecular gas flow lasers. Of particular interest to this study is the Carbon-Dioxide glow-discharge plasma, of which Nitrogen is the principal energy bearing constituent.

The main energy loss mechanisms in a collision dominated plasma are found in the sheath and ambipolar regions. The sheath voltage loss occurs directly as a result of Debye shielding, which creates a space charge layer abutting an electrode or wall. The ambipolar losses are associated with ambipolar diffusion, relating to the charged species density being less than the free-stream or equilibrium density. This loss exists even in the absence of current flow.

An initial premise of probe theories that takes account of collisions between charged particles, atoms and molecules, is that the plasma is quasi-neutral up to a certain distance from the probe and that the electric fields are localized within a narrow sheath region adjacent to the surface of the probe. The problem of making the transition

from the quasi-neutral solution to the sheath solution was usually resolved through matching techniques. The description of a collision dominated sheath together with the quasi-neutral ambipolar region presents an imposing set of coupled nonlinear partial differential equations, for which there exists no known exact solution. This study reports a numerical method that describes the sheath and ambipolar regions in a self-consistent fashion.

In the hydrodynamic approximation (i.e., collision dominated), the dimensions of the space charge region "λ_s" (the sheath length) is larger than the mean free path "ℓ" of the species particles. Additionally the sheath characteristic length is very much smaller than the characteristic length of the plasma boundary layer. This puts our study in the collisionally thin sheath region, [Ref. 1].

However, the plasma is not considered to be in thermal equilibrium. In fact for plasmas in thermal nonequilibrium, the electron temperature is governed by the electron energy equation, which includes the effects of the electron energy-conduction and electric field as well as collisional energy exchange between electrons and heavy-gas particles. The consideration of negligible energy exchange during collisions (thermally frozen) does not generally result in a

constant electron temperature (T_e); particularly in the sheath region, even if other properties such as the ion temperature (T_i) are constant. Therefore the electron energy equation is necessary even in the case of the thermally frozen situation.

An ionizer/sustainer discharge pumped laser requires that relatively high E/n values be distributed as uniformly as possible throughout the bulk of the gas medium. The electrode regions however represent a substantially higher and more inhomogeneous E/n which may impose an instability on the remainder of the discharge. In the region of the electrode, the E/n is significantly higher than the undisturbed plasma. In this region the elevated electron temperature, which is a direct function of the sustainer field, will increase ionization, which could lead to arcing and breakdown.

In the so-called vacuum arc [Ref. 2], anode spots appear under certain operating conditions. These anode spots are of practical importance, having been also observed in high density plasmas [Ref. 3]. They are typically micron or sub-micron sized protrusions on the electrode surface, which act to "focus" the electric field and current stream lines. Primary consideration here is with the anode.

B. REVIEW of PREVIOUS WORK

1. Physics

Most work on sheath phenomena is embodied in "probe" theory investigations, where the effects of the electrode (probe) locally disturb the quiescent plasma. Such work is relevant to this study since the anode is essentially a heavily biased probe in contact with the plasma, which is quiescent within the sheath region.

Analyses of electrostatic probes in collision dominated plasmas were originally done by assuming that a quasi-neutral diffusion controlled region extended to within one mean free path of the probe where it was matched to the edge of the free fall-sheath. No provision was made for a transition region between the two regimes [Ref. 4].

Shultz and Brown [Ref. 5] studied the transition from a collisionless to collision dominated sheath through charged species collection with a spherical probe. Postulating different physical models for cases ranging from a collisionless sheath to a sheath with many collisions, semi-empirical formulas were obtained in each instance. Several other authors attempted matching the solution in the collision dominated region directly to that of the free-fall sheath using variational principles.

The previously mentioned techniques can be criticized, because in each of these methods something was forced to fit. In other words, at least one derivative was discontinuous at some artificially imposed boundary, thereby raising doubt as to the validity of the results.

The first systematic analysis of probes in collision dominated plasmas were carried out by Cohen [Ref. 6] and Su and Lam [Ref. 7], who assumed the continuum equations valid throughout the plasma, including to the probe surface. As a consequence of these analyses, both the sheath and the quasineutral regions appeared naturally from the diffusion equations in the limit as the ratio of probe size to Debye length approached infinity. The continuum model removed the concern about forcing sets of equations to match at the edge of an arbitrarily assumed sheath. These authors restricted the problem to an isothermal plasma, but as will be shown here, the electron temperature may vary significantly in the neighborhood of the probe for most discharges of interest.

Radbill [Ref. 8] quasilinearized the continuum equations, and solved with numerical integration, obtaining results that agreed with those of Cohen where comparison could be made.

In the previous analyses, not only was the isothermal plasma assumed, but the plasma was so slightly ionized that only charge-neutral interactions needed to be taken into account. However, for highly nonequilibrium plasmas, charge-charge collisions play an important role [Refs. 9 and 10]. The effects of collisions between charged particles manifest themselves as volume ionization, volume recombination and the dependence of the transport coefficients on charged particle density and temperature and, therefore on position. Barad [Ref. 11] cites a specific example,

> "If the electrons have a temperature of 2500 °K, then the ionization fraction equal to about 10^{-5} will result in charge-charge collisions occurring one-tenth as frequently as charge-neutral collisions. It is shown... that for such a degree of charge-charge interaction, significant effects are felt."

Chung, Talbot and Touryan [Ref. 1], in an extensive review of probe studies, state that no general solution is available for determining charge density and species temperature for probes small in comparison to the hydrodynamic boundary layer thickness.

Dolson [Ref. 12] investigated the nature and extent of the voltage drops in the vicinity of magnetohydrodynamic (MHD) non-emitting electrodes, in particular the losses attributable to the sheath. Under his conditions the nonexistence of a one-dimensional sheath solution is shown, and a

computer model with two-dimensional, periodic active sites representing a flat plate electrode is developed. With this model the effects of a magnetic field and Joule heating are studied and the results are compared with experimental observations reported by Argryoupolos (1973), and Sonju and Teno (1974), [Refs. 13, 14].

An extension of Dolson's work is reported in 1980 [Ref. 15], for application to ionizer/sustainer discharges. The role of electron pressure is introduced along with net ionization and recombination, which proves necessary in establishing truely boundary-layer behavior for the charged species profiles. Additionally, a modified Newton-Raphson computational procedure, which enables much faster solution convergence is formulated. That study demonstrates solutions with non-active wall densities depressed from the free stream density, a result Dolson did not accomplish. Results were for 10 volts of electrode potential fall, an increase from 5.0 volts reported by Dolson.

2. Computational Methods

The fundamental difficulty in the development of computational models for the prediction of plasma flow phenomena lies in the coupling of equations governing both the gasdynamic flow and the electromagnetic (EM) fields. Since

the flow fields can affect the EM fields (and vice-versa), the nonlinear partial differential equations must be solved simultaneously and the formulation of the problem becomes complicated. In the present study the flow properties are decoupled from the the charged species conservation equation and Poisson's Equation.

The approaches used in the solution of plasma flow phenomena are quickly reviewed below so as to point out the background of the the approach in this study. This problem is typical to those dealing with plasma-boundary layer phenomena. Approaches to solving the general plasma flow problems have consisted of the following;

(a) One dimensional "influence coefficients."

(b) One-dimensional closed form analytic solutions to ordinary differential equations.

(c) Similarity transformation and closed form solutions to a modified differential equation in one- or two-dimensions.

(d) One-, two-, or three-dimensional numerical solution to the uncoupled steady plasma flow phenomena equations.

(e) One-, two-, or three-dimensional numerical solution to the coupled steady plasma flow phenomena equations

using either the magnetic induction equation or the elliptic equation governing the electric fields.

(f) One-, two-, or three-dimensional transient numerical solution to the coupled plasma flow phenomena equations using either the magnetic induction or the elliptic equation.

Methods (a) through (c) are exact but are restricted to a narrow range of problems. Approaches (d) and (e) provide rapid convergence to a steady state solution but do not consider the transient behavior between the gasdynamic and electromagnetic fields, or often the strong coupling between the two sets of fields. Method (f) is most desirable since it treats a wide range of problems, but suffers from large computational times [Ref. 16]. The approach of this study is characterized as a method (e) approach.

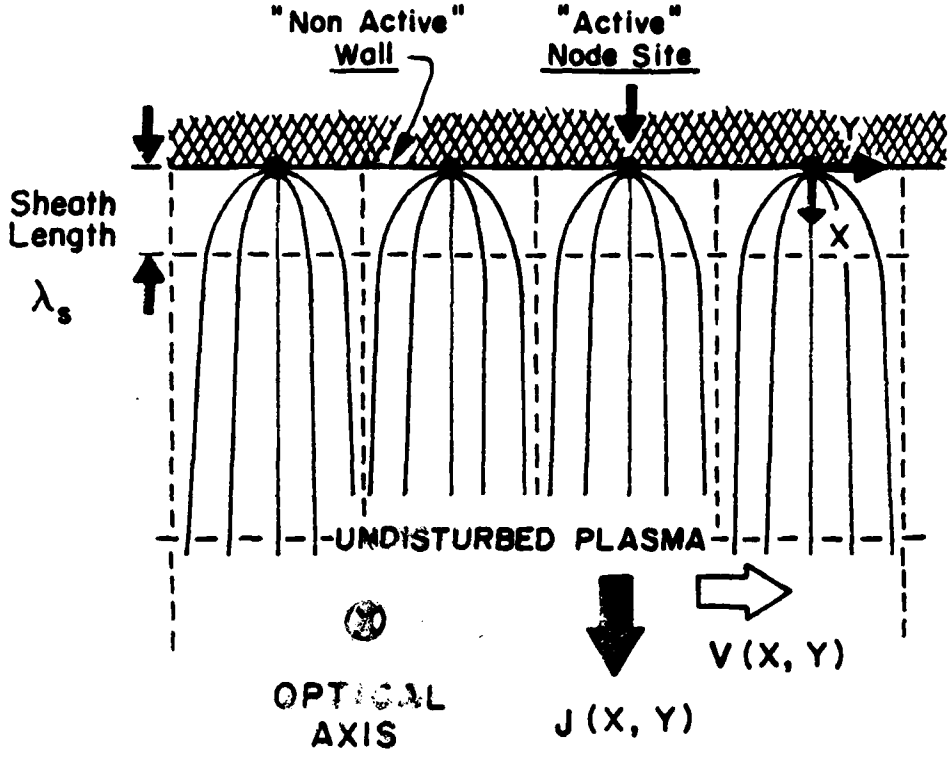
C. OBJECTIVES OF THE PRESENT STUDY

The objective of this study is to demonstrate a numerical procedure which can model the physical behavior of a conducting fluid, as would be generated in the after-glow of an electron beam discharge laser, under the influence of electric forces. Calculations are performed in two dimensions, with a flat plate non-emitting electrode (anode) which has periodic active sites. These active sites

facilitate current constriction required to satisfy charge continuity, Ohm's law, and Poisson's equation. The physical model is idealized in Fig. 1.

The controlling equations are solved for the potential, the charged species density distributions, electron temperature and current in the vicinity of the electrode. The resulting computer programs generate sheath and ambipolar regions in a self-consistent fashion, using the same set of equations throughout the field. The size of the sheath and the voltage drop attributable to its shielding effects are investigated.

This work does not consider those phenomena that lead to the arc discharge, which results from thermal instabilities. The constriction at the anode is purely that required to satisfy continuity.



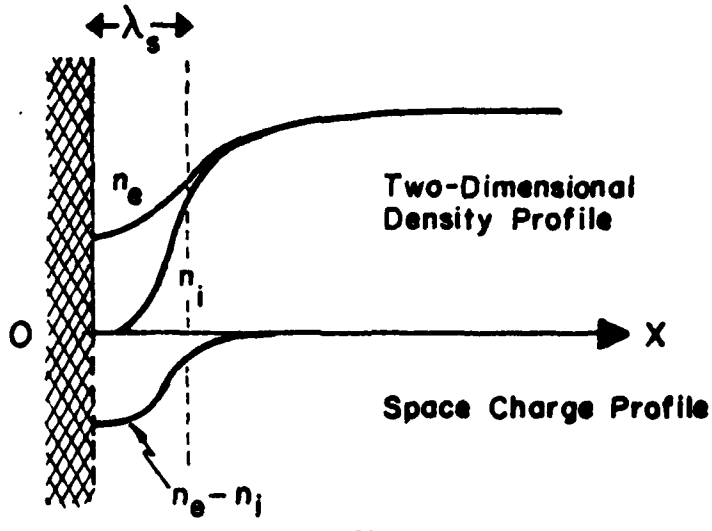


Figure 1

Two-Dimensional Model with Periodic Current Constriction.

II. PROBLEM FORMULATION

A. GOVERNING EQUATIONS

The governing equations that describe the electrode region portion of the plasma, consist of Poisson's equation, species conservation and electron energy equation, in addition to the overall continuity, momentum and energy equations of the plasma. These overall plasma equations are essentially those of the background gas since the plasma is weakly ionized (less than 0.01 %).

We have [Ref. 1];

Poisson equation...

$$\nabla^2\phi = \frac{e}{\epsilon_0}\left(n_e - \bar{z} n_i\right) \tag{1}$$

electron conservation...

$$u\cdot\nabla n_e - \nabla\cdot\left[D_e\left(\frac{n_e}{p_e}\nabla n_e - \frac{e}{kT_e} n_e \nabla\phi\right)\right] = \dot{n}_e \tag{2}$$

ion conservation...

$$u\cdot\nabla n_i - \nabla\cdot\left[D_i\left(\frac{n_i}{p_i}\nabla p_i + \frac{e}{kT_i} n_i \nabla\phi\right)\right] = \dot{n}_i \tag{3}$$

In gas discharges, the electron energy distribution is known to be non-Maxwellian. In order to allow for non-Maxwellian distributions and still retain use of a temperature concept, an empirical fit to the calculated relation of electron temperature as a function of E/n is used in the representation of the electron temperature (see Sec. III.A). The concept of a "two-temperature" plasma is preserved but no restrictions are made on the energy distribution function of the electrons. In addition to the assumption of a functional dependence of the electron temperature upon E/n only, there is no representation of an external magnetic field and, in particular the internal or self-generated magnetic fields are not considered. The overall gas temperature is assumed constant and in equilibrium with the plasma ions. The bulk energy equation is assumed to have little bearing on this study, since there is negligible gas heating as discussed in Section III.A. This effectively uncouples overall energy consideration from the problem.

The electrodes serve the purpose of sustaining the E/n to pump the laser medium. No account is made of an external E-beam or other source of primary ionization. Simply, the medium may be considered to exist in an afterglow state.

The production term (i.e., the right hand side) in Eqns. (2) and (3) has been described utilizing measured and theoretical results of the rate coefficients for Nitrogen ionization and two-, and three-body recombination. The production term is expressed as;

$$\dot{n}_e = \nu_i n_e - \alpha_1 n_e n_i - \alpha_2 n_e^2 n_i - \alpha_3 n_e n_i n_o \qquad (4)$$

where "ν_i" and "α_i" are respectively the ionization and recombination rate coefficients.

B. BOUNDARY CONDITIONS

In the customary boundary-value problem in fluid mechanics, a two-dimensional flat plate is assumed. A similar two-dimensional Cartesian description is used in the present work, which has been found to be the minimum suitable description for the problem posed, [Refs. 3,12,15]. No one-dimensional solution exists for the frozen flow of charges through a collisional sheath, since a Cartesian geometry in one dimension is devoid of the necessary geometric decrease of current density away from the electrode, as there would be in a cylindrical or spherical geometry.

Convection is considered negligible, as can be demonstrated by comparing fluid velocity with the drift velocity.

Electron and ion densities are determined at the plasma edge, knowing the state of the medium. A current balance can be used to establish the species densities resident at the active portion of the anode. An electrode voltage drop is determined from the properties of the medium; e.g., the effective ionization energies of the gas in particular. Since current constricts, there must be regions of the flat plate that receive no current. The inactive portion of the flat plate merits attention, especially in the regions adjacent to the active anode site. Species concentration along the inactive portion of the wall can vary, depending on the reactivity (catalytic properties) of the wall. There is a balance that can be established between the voltage variation normal to the wall, and the species concentration at the wall together with the species variation normal to the wall. A zero concentration of electrons and/or ions at the wall is an acceptable boundary-condition; which is to say that the wall acts as a perfect sink for the charged species. A detailed kinetic-theory analysis shows that there is in fact a lower bound to the electron and ion concentrations at the wall [Ref. 17]. In any case, there is no current into or out of the inactive wall, (see Fig. 1). This is discussed later in Sect. IV.

The remaining portion of the boundary conditions refer to the free stream medium, and are easily established.

C. PARAMETERS OF THE DISCHARGE MEDIUM

The application of this study is for molecular Nitrogen at one amagat, where the gas temperature is 273 °K. Other pertinent parameters follow;

Electric Field: 1.5 to 15 KV/cm.

Electron Density: 10^{+17} to 10^{+19} m^{-3}.

Electrode Voltage Drop: 35 volts.

The electron energy relation is, as mentioned, a curve fit from experimental results [Ref. 18], carried out by R.W. Compton and D.J. Sutton (1952). Since cross-section data for electrons in Nitrogen discharges are available [Ref. 19], the electron temperature and diffusion coefficient are readily determined, and representable as a function of E/n.

The ion temperature is assumed to follow that of the neutral gas. An electrode potential fall of about 35 volts may be assumed since it is the effective ionization energy for Nitrogen gas and, therefore, the anode fall [Ref. 20].

The cathode is more complicated to describe because of the requirement of surface electron emission. The cathode fall in cold cathodes is of the order of 250 volts, and is considerably less in thermionic cathodes. A proper

III. PROBLEM SOLUTION

A. PROBLEM SIMPLIFICATION (PHYSICS)

There are a number of simplifications that are incorporated in the problem description. Namely;

- Steady state conditions (within the pulse time).
- No magnetic fields.
- Negligible convection.
- Negligible Joule heating, $T_i = T_e$ = constant.

However, specific account is made of;

- Ionization/recombination.
- Electron pressure.

These are discussed in part in this section.

1. Convection

The ambipolar region is the transition region from the sheath to the undisturbed plasma and can perhaps span the boundary layer. This fact requires some consideration of the presence of convective effects.

From the ion species conservation equation (Eq. 3), for example, we know that in the ambipolar region the last term of the left hand side is small compared to electric conduction. Now we need only to compare the convection with

the conduction, or simply the fluid velocity with the drift velocity.

$$v = V_\infty R^{-1/2} \approx 0.1 \text{ to } 1.0 \text{ m/sec}$$

where the Reynolds number (R_e) is 10^4 to 10^6 and the free stream velocity is 10 to 100 m/s. Now the drift velocity may be approximated by

$$v_d \approx \mu E_\infty \approx 10 \text{ to } 100 \text{ m/s}$$

for $E = 10^5$ to 10^6 V/m and the mobility $\mu = 10^{-4}$ $m^2/s/volt$. Clearly, in the presence of a relatively strong interelectrode field, the contribution of ion convection due to a cross flow should be negligible. The geometrical orientation of the discharge with respect to the primary flow can thus become an important consideration.

2. Density Boundary Layer Formation Time

The charged-species density profiles change in a more complicated manner than either the voltage potential or the electric field. The magnitude of the ion and electron densities may change appreciably within the sheath; moreover, fractional analysis is risky because inflections are present in the profiles. A discussion pertaining to the stability of the ambipolar region and its boundary layer nature is presented in detail by Biblarz, et al., [Ref. 15]. However, Appendix A shows that the characteristic length of

the ambipolar region is of the order of the characteristic sheath length. Letting $L=5\times10^{-5}$ meters and the drift velocity as before, v_d = 10 to 100 m/s, a characteristic time for the formation of the density boudary layer would be 0.5 to 5.0 microseconds. It is reasonable that the ambipolar diffusion to the wall can establish itself well within the 50 microsecond pulse period of the electrodes.

3. Significance Of Joule Heating

For the present problem, joule heating can be shown not to be significant. In the vicinity of the active site, the average energy deposition rate per unit volume is shown to be approximately 4×10^{8} watts/m^3. The temperature of the Nitrogen gas at one amagat would be raised approximately 1°K in 2.6×10^{-6} sec. Allowing a 50×10^{-6} sec pulse period would raise the temperature of that region by approximately 20°K, or 7%.

Assume that the heat generated at the anode spot is carried away by thermal diffussion (conduction). Then for Nitrogen gas (k=0.0267 W/m-°K, ρ =1.138 kg/m^3, and c_p =1043 J/kg-°K) the thermal diffusivity is;

$$\alpha = \frac{k}{\rho c_p} = 2.25\times10^{-5} \text{ m}^2/\text{sec}$$

A typical diffussion time would be (for, λ_j =4.4×10^{-5} m);

$$\tau_d \simeq \lambda_j^2/\alpha \simeq 0.86 \text{ millisec}$$

The "flush time" is the time required for clearing the lasing cavity of residue gases in preparation for the next pulse, and can be estimated as the gas flow rate divided by the flow length. Assuming v = 100 m/sec and L = 10 cm, the flush time would be;

$$FT = 0.1/100 = 1 \text{ millisec}$$

Other factors considered, the inter-pulse period (time between pulses) would be larger, but even in a one millisecond period, there is just enough time to cool the anode spot.

The analysis so far has assumed a flow velocity at the surface to be the same as the bulk flow in the laser cavity. However a boundary layer exists-- probably turbulent, with a laminar sub-layer adjacent to the electrode for;

$$R_e = xV_\infty/\nu = (0.1)(100)/(15.53\times10^{-6}) = 6.44\times10^5$$

The velocity boundary layer thickness would be 2.55 millimeters. Assuming a 1/7-th power profile, the velocity at a sheath length from the electrode would be:

$$v = V_\infty(\lambda_s/\delta)^{1/7} = 100(4.4\times10^{-5}/2.55\times10^{-3})^{1/7} = 56 \text{ m/sec}$$

A characteristic local flush time would be approximately 2 milliseconds, and much higher for a laminar velocity profile. These flush times can be quite large as the requirement to cool the Joule heated gas near the anode spot is

increased. The design flush time based on other factors may infact be insufficient to clear enough of the Joule heated gas from the vicinity of the anode spot, enabling a build up of heated gas, which could lead to local enhanced ionization of the gas.

4. Electron Temperature

The electron temperature (T_e) is assumed to be a linear function of E/n as presented in Fig. 2, for the region of interest, namely E/n = 0.5 to 5.0×10^{-20} V-m^2. For the range of E/n considered, it is adequate to take the electron temperature as;

$$T_e = [A \cdot \log(E/n) + B] \cdot T_o \qquad (5)$$

where;

A = 12.1

B = 38.5

E/n is normalized to 10^{+20} V-m^2

The diffusion coefficients were determined from Brown [Ref. 18]. The drift velocities were obtained from plotted results, and since $\mu_s = v_d / E$, and $D_s = \mu_s k T_s / e$, (Einstein's relation), the diffusion coefficients are;

$$D_e = 0.0665 \text{ m}^2/\text{sec}$$

$$D_i = 8.0 \times 10^{-6} \text{ m}^2/\text{sec}$$

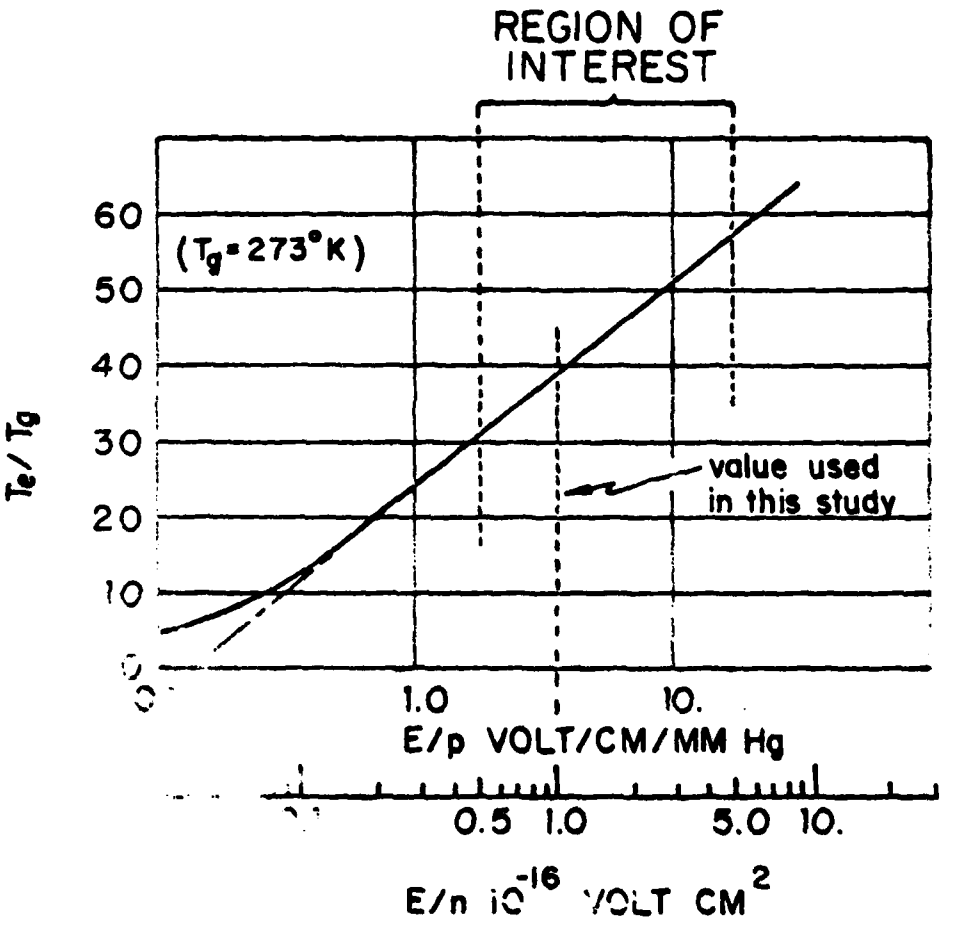


Figure 2

Ratio of Electron - Nitrogen gas Temperature

(Ref. 18)

B. WORKING EQUATIONS

Since convection is negligible, and assuming that the sustainer operates in the afterglow of an electron beam, the governing equations, namely Eqs. 1,2 and 3, can be written as follows;

$$\nabla^2\hat{\phi} = c_1(\hat{n}_e - \hat{n}_i) \tag{6}$$

$$-\hat{\nabla}\cdot[\hat{\nabla}\hat{n}_e + \hat{n}_e\hat{\nabla}\theta/\theta - \hat{n}_e\hat{\nabla}\hat{\phi}/\theta] = c_2\hat{n}_e \tag{7}$$

$$-\hat{\nabla}\cdot[\hat{\nabla}\hat{n}_i + \beta\hat{n}_i\hat{\nabla}\hat{\phi}] = c_3\hat{n}_e \tag{8}$$

where;

$$\hat{n}_s = n_s/n_{e\infty} \qquad \hat{\phi} = e\phi/kT_{e\infty}$$

$$\theta = T_e/T_{e\infty} \qquad \beta = T_{e\infty}/T_0$$

$$c_1 = e\phi/kT_{e\infty} \qquad c_2 = \lambda_s^2 n_{e\infty}/D_e \qquad c_3 = c_2 D_a/D_i$$

The equations may now be applied to flat plate formulation as depicted in Fig. 3. It is convenient to drop the "^" for simplicity, but it is to be understood that all variables have been suitably non-dimensionalized.

We have Poisson's equation;

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = c_1[n_e - n_i] \tag{9}$$

electron conservation equation;

$$c_1 n_e(n_e-n_i) + \left[\frac{\partial n_e}{\partial x}\frac{\partial \phi}{\partial x} + \frac{\partial n_e}{\partial y}\frac{\partial \phi}{\partial y}\right] - \frac{n_e}{\theta}\left[\frac{\partial \phi}{\partial x}\frac{\partial \theta}{\partial x} \quad \frac{\partial \phi}{\partial y}\frac{\partial \theta}{\partial y}\right]$$

$$-\left[\frac{\partial \theta}{\partial x}\frac{\partial n_e}{\partial x} + \frac{\partial \theta}{\partial y}\frac{\partial n_e}{\partial y}\right] - \theta\left[\frac{\partial^2 n_e}{\partial x^2} + \frac{\partial^2 n_e}{\partial y^2}\right] \qquad (10)$$

$$+ \frac{n_e}{\theta}\left[\left(\frac{\partial \theta}{\partial x}\right)^2 + \left(\frac{\partial \theta}{\partial y}\right)^2\right] - n_e\left[\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right] = c_2 \theta \dot{n}_e$$

ion conservation;

$$c_1 \beta n_i(n_e-n_i) - \beta\left[\frac{\partial n_i}{\partial x}\frac{\partial \phi}{\partial x} + \frac{\partial n_i}{\partial y}\frac{\partial \phi}{\partial y}\right] - \left[\frac{\partial^2 n_i}{\partial x^2} + \frac{\partial^2 n_i}{\partial y^2}\right] = c_3 \dot{n}_e \qquad (11)$$

C. NUMERICAL METHODOLOGY

The nonlinear terms creat problems in the computer analysis of the electron and ion species conservation equations, (Eqs. 10,11). The Jacobi method includes all nonlinear terms on the "right hand side"; i.e., external to the coefficient matrix. Convergence to a solution is possible if these non-linear terms change slowly over each iteration. It was found by Dolson [Ref. 12] that the Jacobi method was

in fact unstable for the present set of equations and conditions. As a consequence of the failure of the Jacobi method, a quasi-Jacobi method was implemented, in which an estimate for each of the solution variables (ϕ, n_e, n_i) was computed. When the product of two variables is encountered, one variable is treated as a constant coefficient for each iteration. This means that the non-linear terms are retained in the coefficient matrix. The "constant" coefficients are updated after every iteration, thus altering the coefficient matrix. The conventional Jacobi method was found to converge only for low values of voltage potential at the electrode, whereas the quasi-Jacobi procedure provided converged solutions for approximately 5 volts. A discussion of these methods is provided in Appendix C as background for the development of the modified Newton-Raphson (MNR) method, which is used in the solution to Eqs. 1 through 4.

The nonlinear coupled partial differential equations (PDE), Eqs. 9,10 and 11 are linearized to provide a set of linear algebraic equations, which must hold throughout the domain described as follows. The domain as illustrated in Fig. 3, is divided up into equal sided sub-areas whose vertices are the computational nodes at which the linearized

algebraic equations must hold. The wall is represented by seven (7) nodes, with the leftmost node being the location of the active site. The "y" direction extends from this active site along the wall. The "x" dimension perpendicular to the wall is represented by 25 nodes starting from each of the wall nodes. The solution to the PDE's is represented by the vector $z(x,y)$, or on the computational mesh as $z(i,j)$.

The boundary conditions have been discussed previously and are presented in Fig. 3. The boundary condition that there is no net current into the nonactive wall ($J_x = 0$) is satisfied by the electron contribution to current only. This is justified since it can be shown that the electron contribution is approximately three orders of magnitude greater than the ion contribution to the total current.

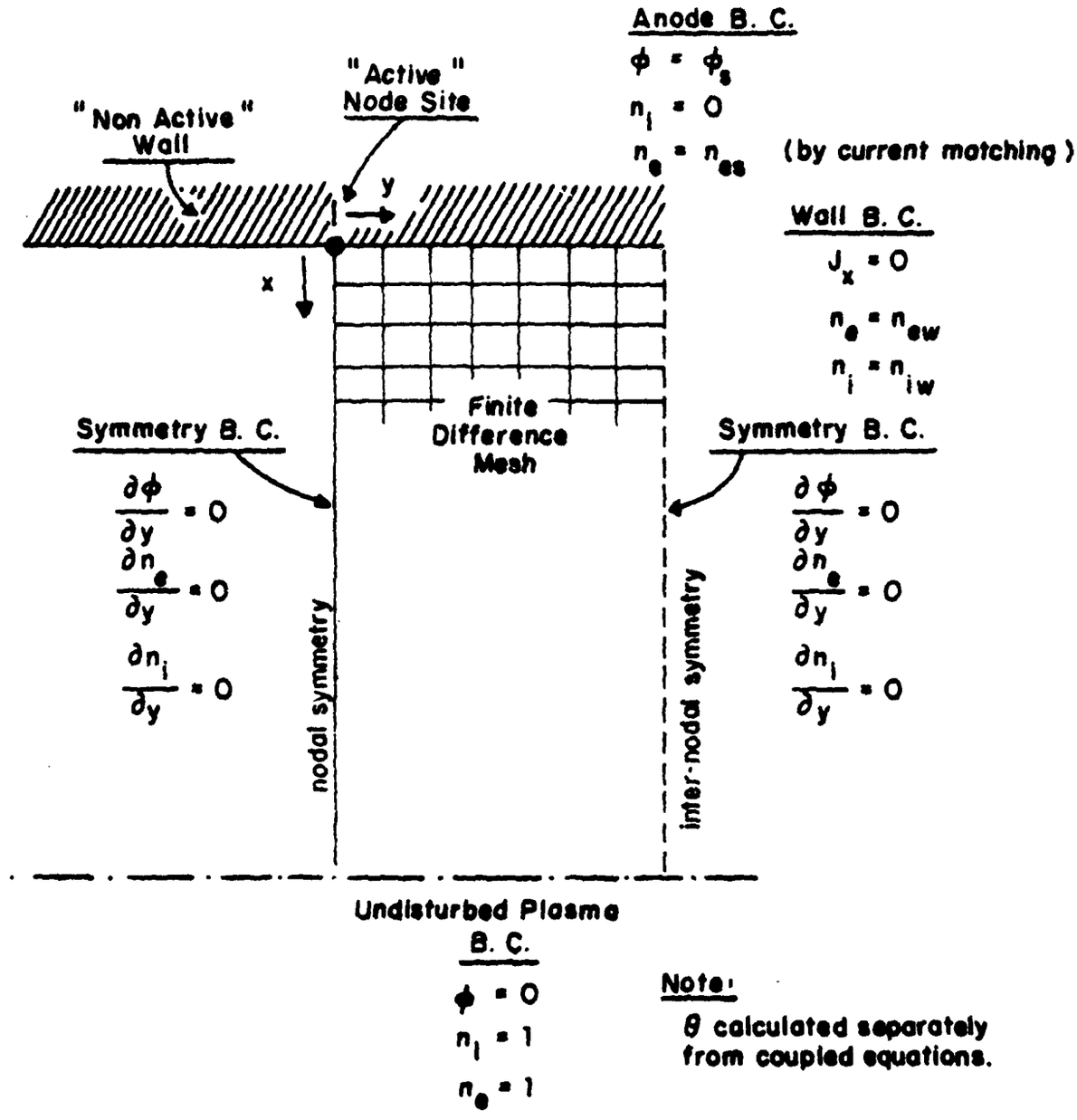


Figure 3

Boundary Conditions for Computational Method.

IV. RESULTS AND CONCLUSIONS

A. GENERAL DISCUSSION

Computer solutions are reported here for three cases presented in Table I. The active site voltage was 35 volts in each of the three cases. Included in this section are

TABLE I: Computer Solution Conditions

CASE	n_e (wall)	n_i (wall)	n_e (anode)	n_i (anode)
I	1.0	0.0	J-matched	0.0
II	"float"	0.0	not J-matched	0.0
III	"float"	"float"	not J-matched	0.0

also some pertinent results obtained at 10 volts to illustrate the effects the net production term has on the convergence to a boundary layer behavior for the species density and free stream electric field.

The solutions are depicted as oblique presentations of the potential, species density, space charge, net production, electric field and Joule heating distributions over the "x-y" coordinates defined in the problem formulation. Also included are two-dimensional plots of various terms along a "cut" or line perpendicular to the electrode wall,

starting at the active site, with some starting at the wall along the inter-nodal line of symmetry (see Fig. 3). The solutions are organized as Case I,II and III results, will be presented as Figs. 12,13 and 14 respectively at the end of this section. The discussion of these results proceeds with additional figures which are interspersed in the text appropriately.

Table I presents various electron and ion boundary conditions along the wall some of which are "floated." By this term is meant that upon the completion of each numerical iteration, the electron (and/or ion) density at the nonactive portion of the wall is re-established by assigning it the value of a near neighbor away from the wall. This process is illustrated in Fig. 4.

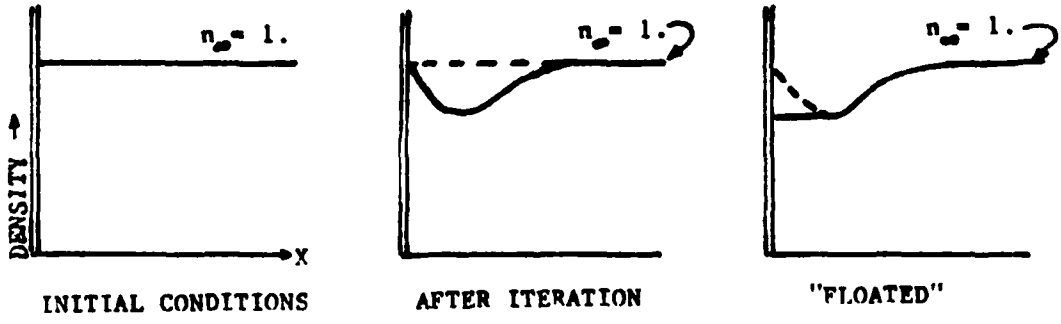


Figure 4

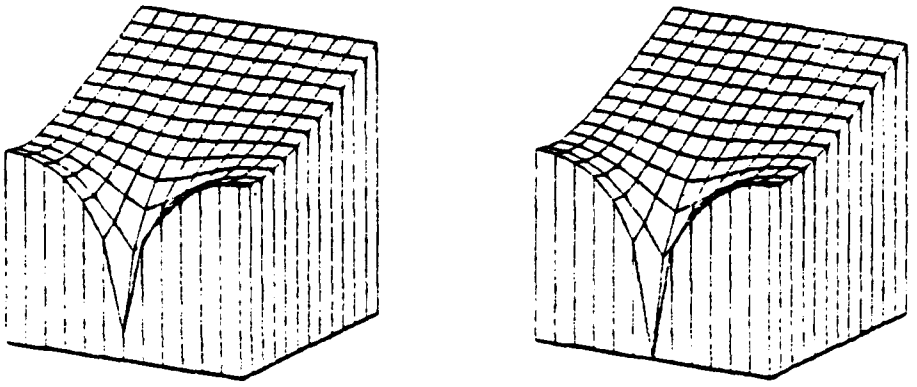
Description of "floating the Wall Boundary Condition.

The usual elliptic boundary value problem has well possed boundary conditions from a priori information. However, the nature of the problem in this study includes very little information as to the correct species wall density. It seems reasonable to set these densities to zero along the inactive portion of the electrode as previously mentioned in Sec. II.B. In this study the wall boundary conditions are "lossened" to facilitate "naturally occuring" and unpresuming species density profiles. The nature of the problem in effect, produces a self-consistent boundary condition.

B. EFFECTS OF NET PRODUCTION

Earlier work [Ref. 15] reported limited results for a 10 volt active node site. Solutions for cases with and without the inclusion of the net production term have profound influences on the appearance of the charged species density profiles. Figures 5,6 and 7 illustrate the before and after effects of including the net production term. In both cases the wall boundary values for the species density was initialized at the free-stream value (i.e., 1.0), then allowed to "float" as described previously. The addition of the net production term brings about the boundary layer behavior of the densities of the electrons and ions is illustrated in Fig. 7. Efforts to attain converged solutions with the net

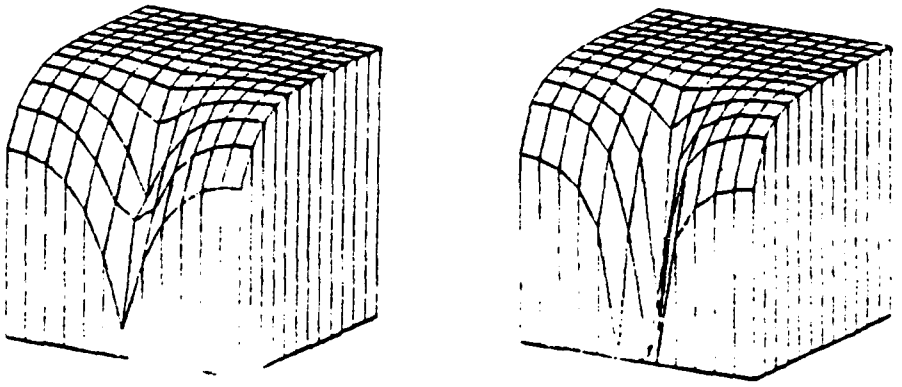
production term set to zero proved fruitless for active site potentials exceeding 10 volts.



Electron Density
(a)

Ion Density
(b)

Figure 5
(a) Electron and (b) Ion density perspectives with the net production term set to zero. (10 volts)



Electron Density

Ion Density
(b)

Figure 6
(a) Electron and (b) Ion density perspectives with the net production term coupled. (10 volts)

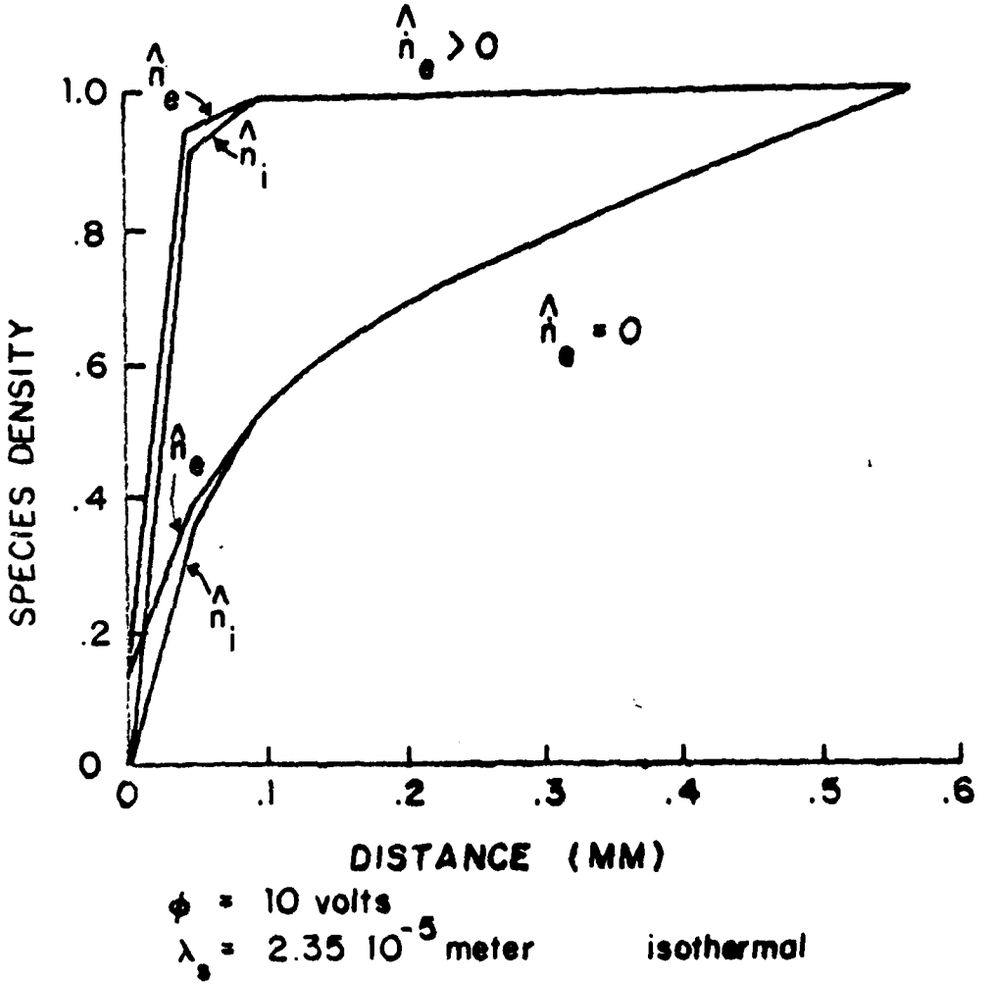


Figure 7

Influence of the net production term on the density profiles along a line perpendicular to the wall, extending from the active node site, (10 volts).

C. DISCUSSION OF RESULTS

1. Voltage Potential

The voltage potential of the active node site ("the anode") in each of the cases presented was 35 volts. Figures 12-a, 13-a and 14-a will clearly illustrate sharp voltage spikes at the anode, and the "Laplacian" nature of the field far from the anode. The potential along the inactive wall varied from 12 volts at the farthest position from the anode to 17 volts just next to the anode. Similar profiles of the electric potential along the wall for the other cases are presented in Fig. 8. Figures 12-i, 13-i and 14-i will show the voltage potential variation along a line normal to the wall extending from the anode site.

Figure 9 illustrates a two-dimensional cut of the voltage potential along the inter-nodal symmetry line. Here it can be seen that the variation from the wall is shallow. It is important to point out that the electron temperature is directly related to the gradient of the voltage potential through Eq. 5. This fact has certain consequences which are discussed in the next section.

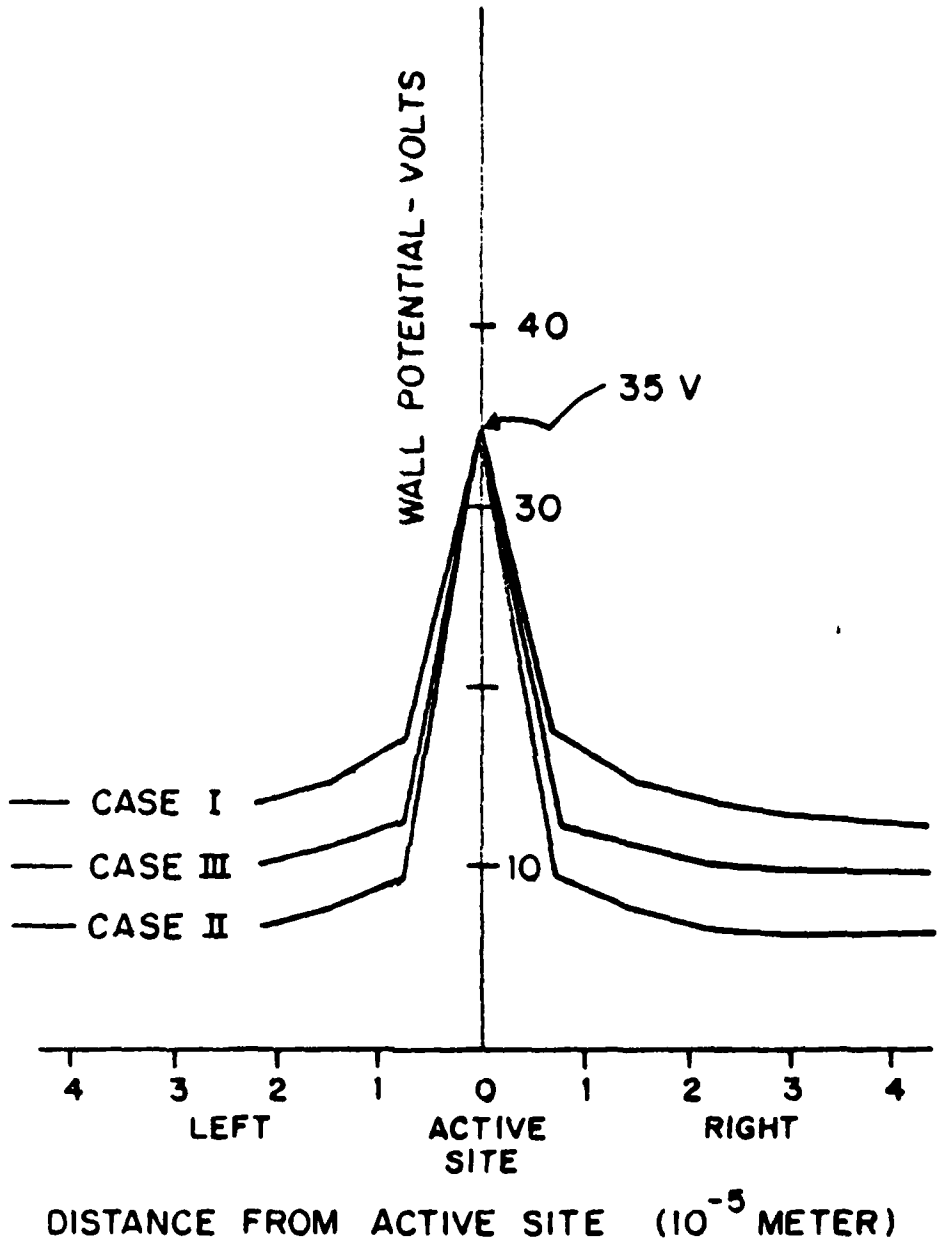


Figure 8

Voltage potential along the wall boundary (electrode) for CASES I, II, and III.

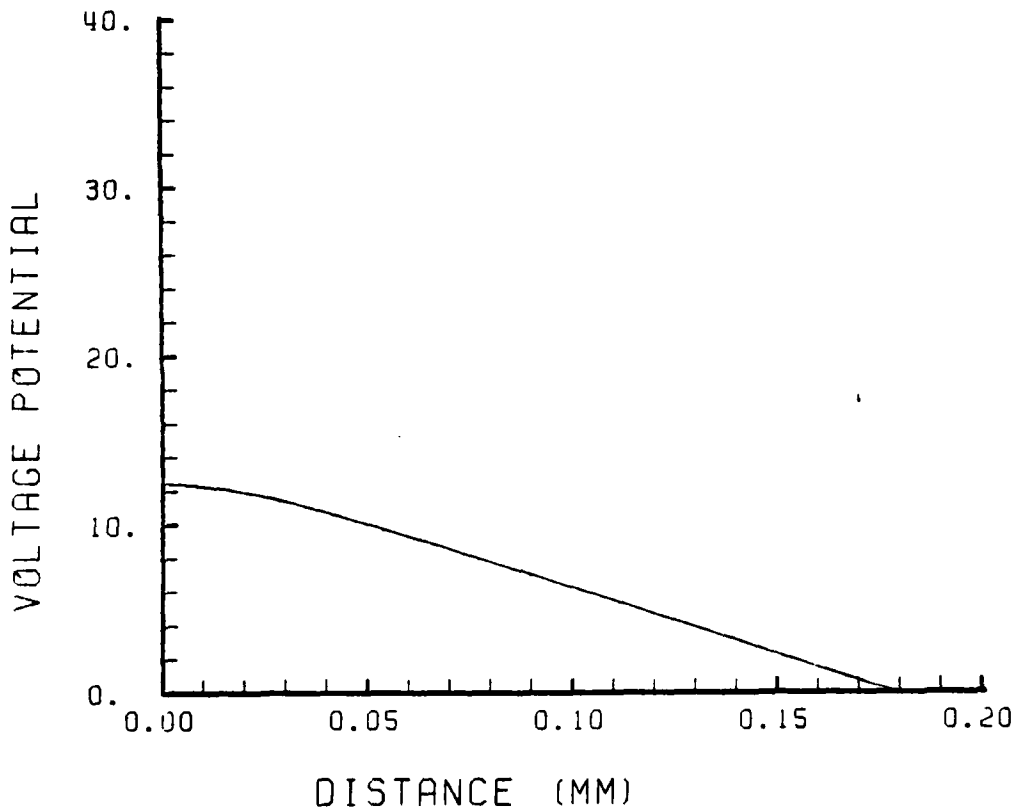


Figure 9

Voltage potential variation normal to the wall, along the inter-nodal symmetry line, (CASE I)

2. Electron Energy

The electron energy will be presented in series -b of Figs. 12,13 and 14, as oblique perspectives. The anode site temperature for Cases I, II and III are 2.1, 2.1 and 1.9 eV respectively. The description of the electron temperature is that of a linear function of the logarithm of the electric field. Any portion of the potential field that is "level" (grad ϕ =0) will necessarily generate a low or even negative (see Fig. 14-b) electron temperature. According to Fig. 2 and the discussion in Sec II.A.4, a negative temperature would occur below an E/n of 0.04×10^{-20} V-cm^2. This problem can be removed by setting a minimum value to the non-dimensional temperature field, nominally 0.13, corresponding to the minimum at the low end of E/n in Fig. 2.

The parameters of the discharge (Sec. I.C) require an electric field of 2.7×10^5 V/m (or, log[E]=5.43) at the free stream boundary, which corresponds to an electron temperature of 0.905 eV (θ=1.). The temperature fields in all cases presented do not quite meet the free stream boundary requirements. The electric fields are insufficient to render proper results to the electron temperature profiles. Thus the free stream temperature boundary requirements are "forced" by slightly inflating the influence of the electric

field. The effect of forcing the temperature field in this way can also be looked at as "sliding" the linear representation of the temperature curve to higher values at constant E/n, (see Fig. 2). Figure 10 illustrates a case where a low electric field, here 0.8 kV/cm, results in a low electron temperature projecting toward the free stream (θ=0.62). The low temperature is adjusted to meet the desired free stream value (θ=1.0) by multiplying the electric field by a factor of 3.40. Figure 10 also illustrates an alternate method of meeting the free stream conditions by truncating the variation of temperature with a minimum value, namely the free stream boundary condition.

Both of the "forcing" techniques are used, but for different reasons. As long as the adjustment factor does not exceed an order of magnitude change in the electric field, then it is not objectionable in light of the simplified representation of the electron temperature. The adjustment requirements of the three cases reported in this study are between 3.0 and 5.0, with Case I using a factor of 10 to see the effects on the Joule heating. In Case I the non-dimensional temperature is artificially high at 1.4 eV at the free stream.

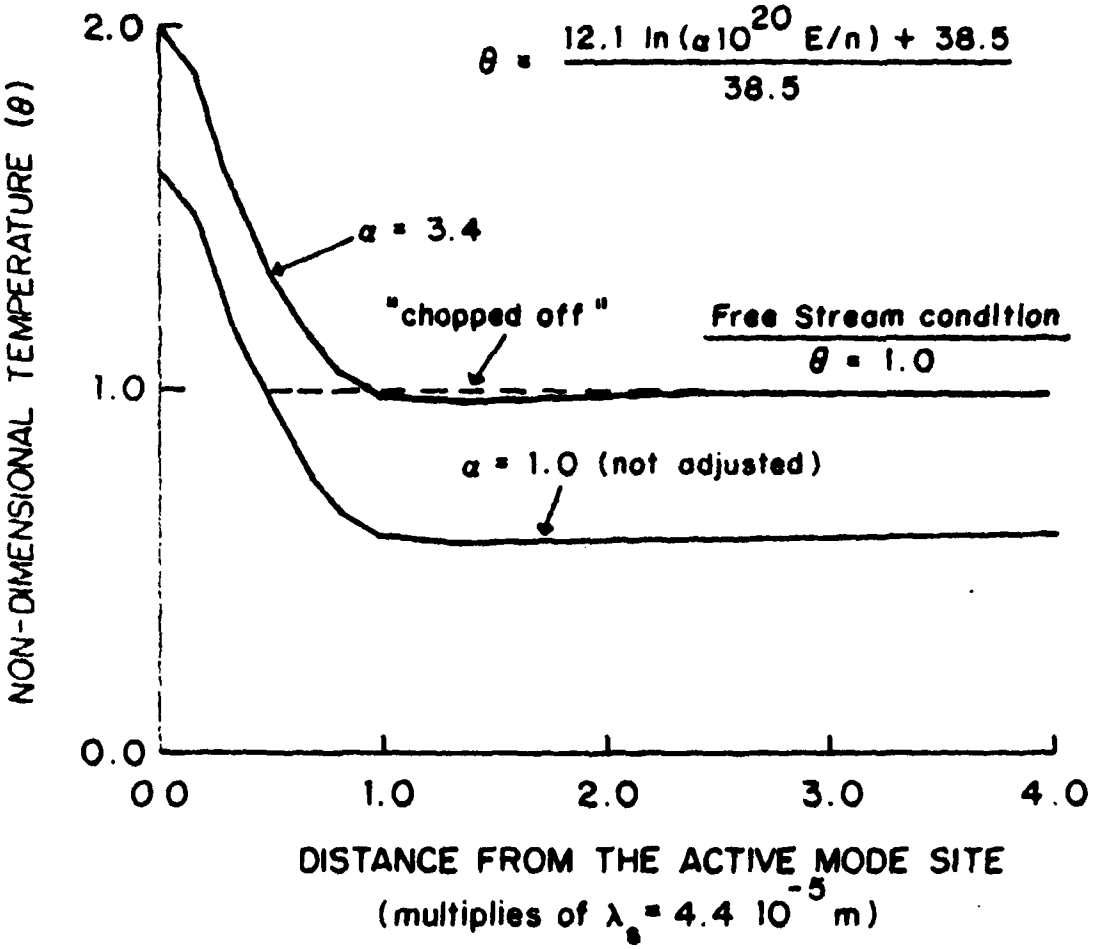


Figure 10

Electron temperature variation normal to the wall, along the nodal symmetry line; illustrating two methods of adjustment to "force" the free stream temperature conditions.

The electric field near the inter-nodal symmetry points of the wall are characteristically 2 orders of magnitude less than that of the free stream conditions. The resulting very low, even negative temperatures in Case III, Fig. 14-b, is "patched" using the truncation method previosly discussed. Whether these low temperatures and the associated steep derivatives are physically reasonable, merits further investigation.

The temperature fields generated and presented as Figs. 12, 13, 14 series -b are the final result depictions. However, the temperature fields used in the solution process were adjusted and truncated as previously discussed to maintain solution convergence. An example of a "corrected" temperature field is presented in Fig. 14-c.

3. Electric Field And Joule Heating

The electric field and Joule heating distributions are generated from the primary voltage and electron density solutions, much in the manner as the temperature distribution was determined. In fact, the Electric field and the Joule heating distributions are similar in appearance to the temperature distributions. These distributions play no direct part in the solution scheme and thus being ancillary in nature were not "forced" to meet any a priori conditions.

The Joule heating term can be written as;

$$J \cdot E = [\, D_e \, n_{e\infty} k T_{e\infty} / \lambda_D^2 \,] \cdot [\, \frac{n_e}{\theta} \nabla\phi - \nabla n_e \,] \cdot \nabla\phi$$

and thus is a rather straight-forward function of ϕ, n_e, and n_i. The ion contribution is shown to be negligible, since $D_e >> D_i$. The Joule heat at the anode site is of order 10^8 and is 100 times that of the free stream values. Joule heating effects are negligible in the overall problem, as discussed in Sec. II.A.3, obviating the requirement for an energy equation.

4. Species Density Distribution

The solutions for the charged species density distributions are presented as oblique perspectives in Figs. 12,13 and 14, series -e (electron density); series -f (ion density); and series -g (space charge density). Additionally the variation of these distributions normal to the wall along the active node symmetry line are presented in the series -j and -k figures. The oblique perspectives of the densities are reversed for clarity.

The electron and ion densities at the nonactive wall are a subject of interest in the literature, to which little information has been gathered on a firm basis. The results of this study suggest a slightly positive space charge at the inactive wall. Scrutiny of Case III results reveals, a

2.2×10^{16} m^{-3} positive space charge number density, (or 3.5 milli-coulombs). Recall that Case III involved "floating" both of the charged species densities at the wall. In fact, close inspection of Case I and II reveals a positive space of similar magnitude but displaced slightly from the wall. The wall in these cases was set with a negative space charge as the boundary condition. The positive space charge along the wall is most predominant in the vicinity of the inter-nodal symmetry line, and whether the low temperature values in this region have significance, remains to be determined.

The active node site (anode) electron density in Case I (2.4×10^{17} m^{-3}) was determined so as to match the free stream current to the current constricting at the anode. In the results of Cases II and III the anode electron densities are chosen so as to provide a smooth variation extending outward from the active site.

5. Ionization/Recombination

The net ionization term is utilized in all cases; and was found necessary for (1) establishment of the charged particle density distributions, and (2) solution convergence for sheath lengths spanning more than one computational node spacing. The net production term is depicted in Figs. 12,13 and 14 series -h. There is net ionization throughout the

solution domain. However as Case III demonstrates, there is a factor of 5 orders of magnitude greater ionization within the sheath region surrounding the anode site than at the free-stream edge. Cases I and II show ionization along the nonactive wall, simply because the ion density is zero, eliminating significant recombination, (see Eq. 4).

6. Current Description

Figure 11 is typical of the constricting nature of the current stream lines. The potential, electron density and the electron temperature are used to determine the "x-y" components of the current density. Using a "nearest neighbor" influence principal, a local derivative can be determined; from which a streamline can be "marched-out" in small increments. The profiles of all three cases are similar, the fact being that the current matching discussed previously will not affect the current streamline profiles. Figure 11 clearly demonstrates the satisfaction of the current constriction requirement necessary to permit solutions to the system of equations.

The current into the anode site in Case I is matched to the current entering at the free-stream boundary. This specifies the electron density at the anode site. However, in Cases II and III the current at the anode was only three

times the free-stream current. The current matching problem does not deserve serious complaint, since the problem description is simplified to two-dimensions and the current constriction is only required to satisfy continuity.

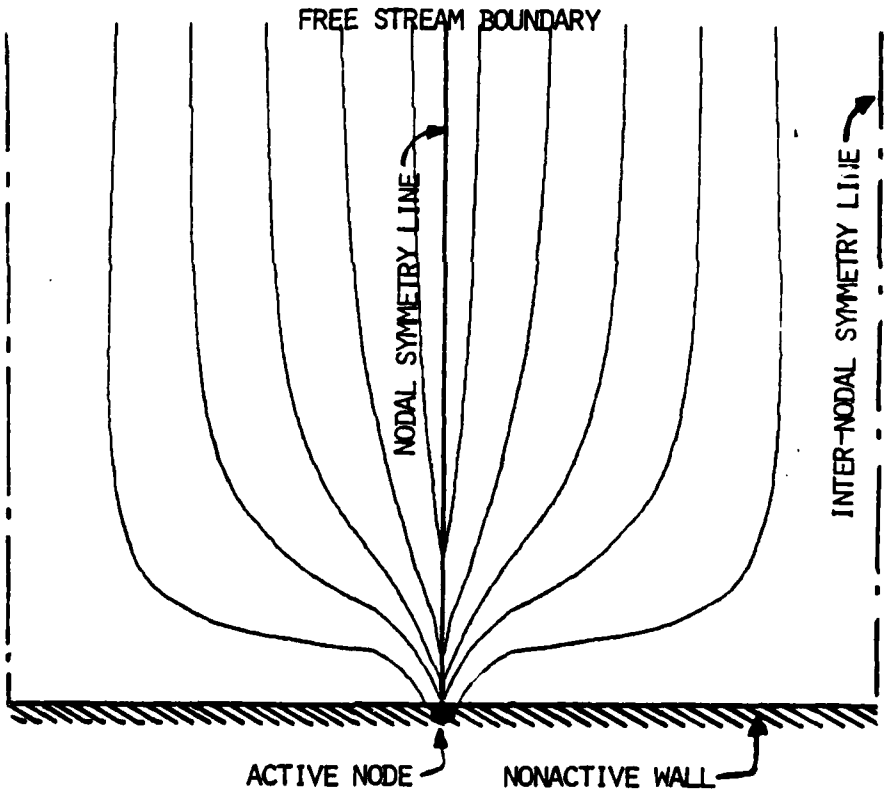


Figure 11

Two-dimensional depiction of the current stream line constriction toward the active node site.

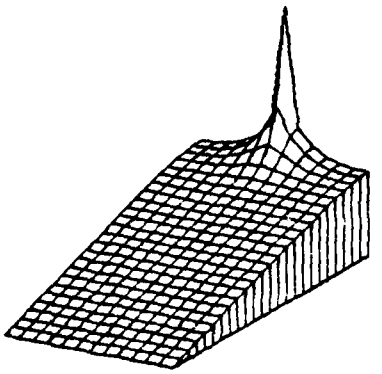


Figure 12-a

Oblique perspective of Potential Field (CASE I)

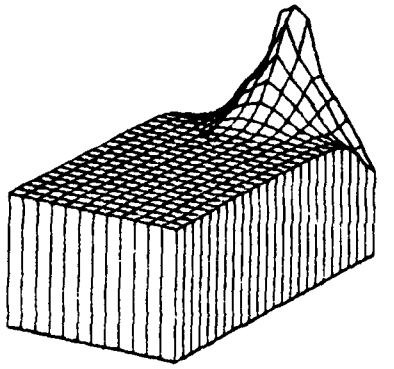


Figure 12-b

Oblique perspective of Temperature field (CASE I)

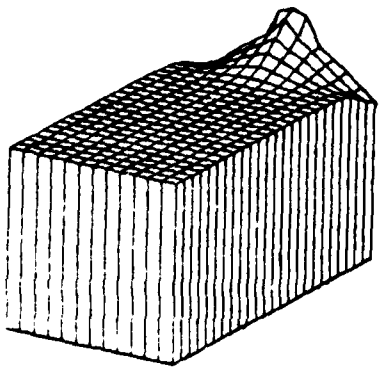


Figure 12-c

Oblique perspective of log Electric Field (CASE I)

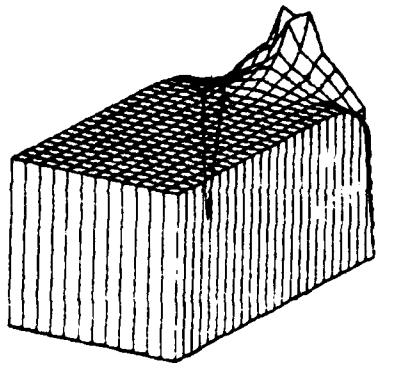


Figure 12-d

Oblique perspective of log Joule Heat (CASE I)

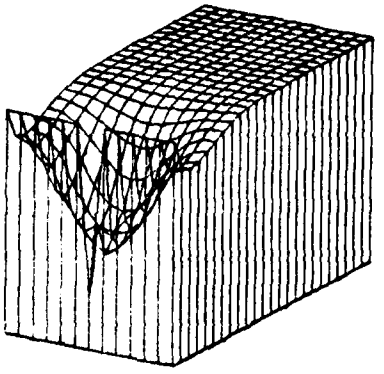


Figure 12-e

Oblique perspective of Electron Density (CASE I) (reversed perspective)

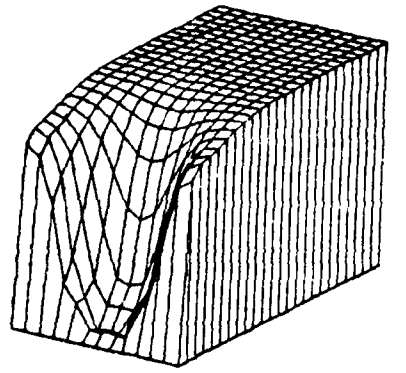


Figure 12-f

Oblique perspective of Ion Density (CASE I) (reversed perspective)

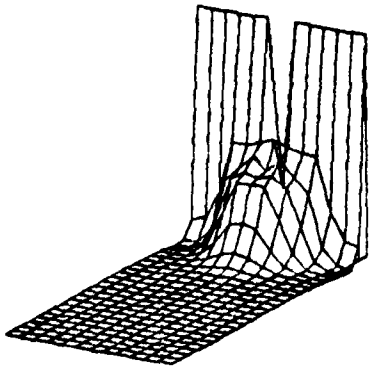


Figure 12-g

Oblique perspective of Space Charge (CASE I)

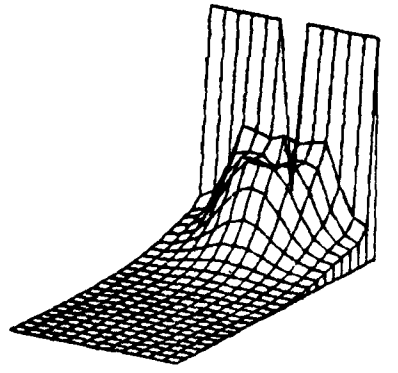


Figure 12-h

Oblique perspective of Net Production (CASE I)

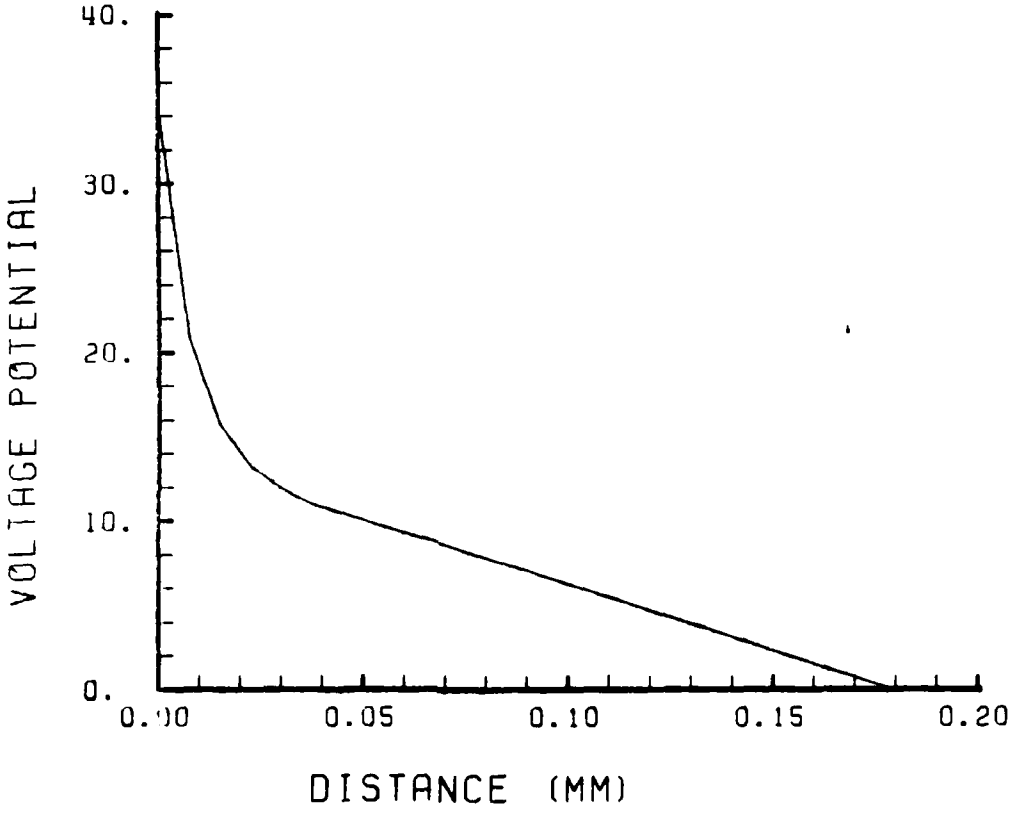


Figure 12-i

Presentation of the Voltage Potential (CASE I) along a line normal to the wall, extending from the active node site.

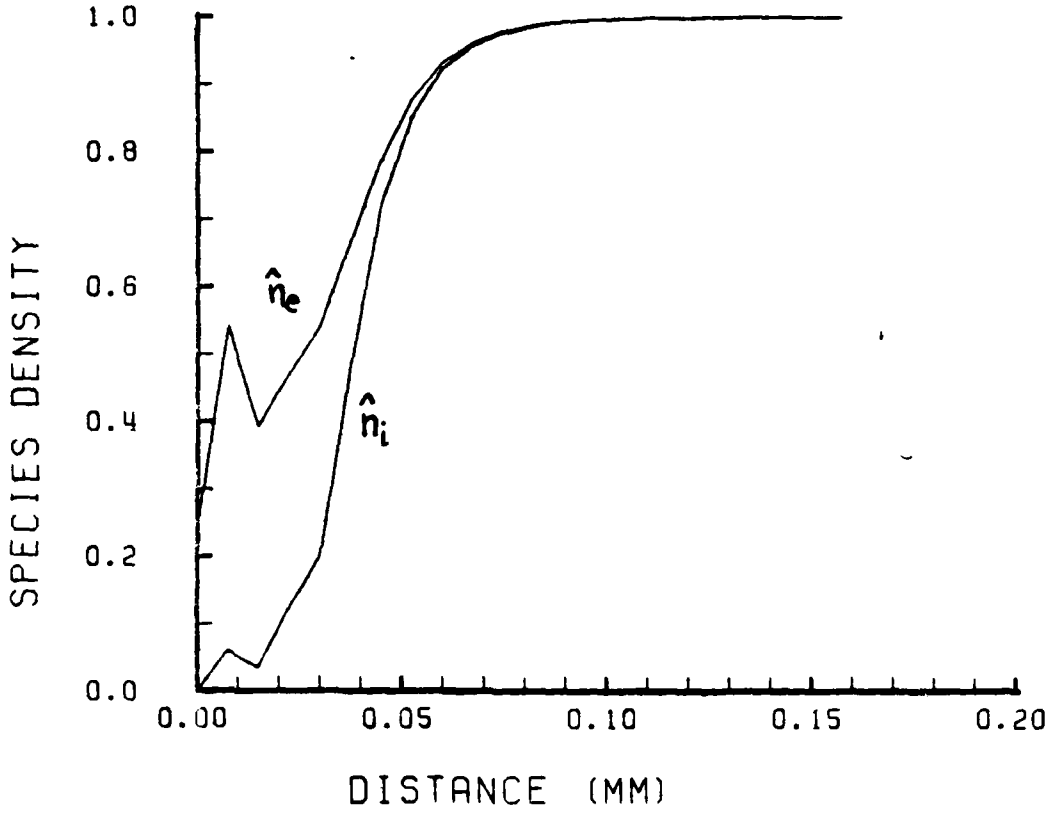


Figure 12-j

Presentation of Species Density (CASE I) along a line normal to the wall, extending from the active node site.

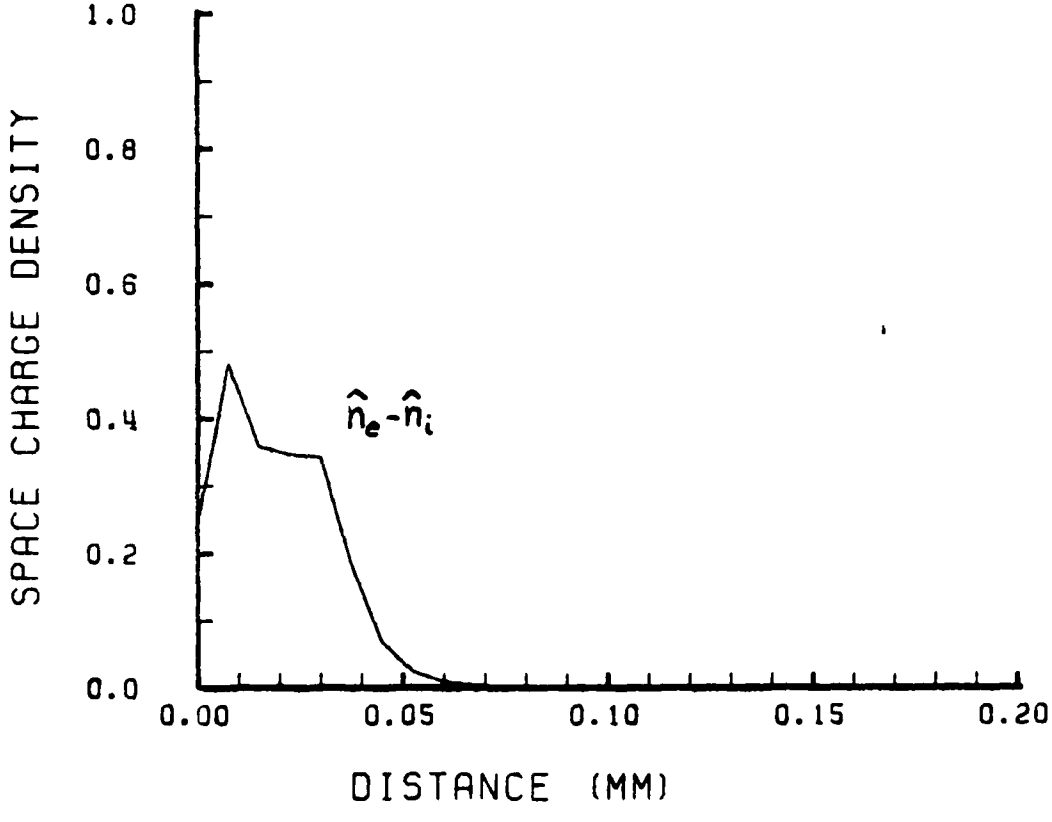


Figure 12-k

Presentation of the negative Space Charge Density (CASE I) along a line normal to the wall, extending from the active node site.

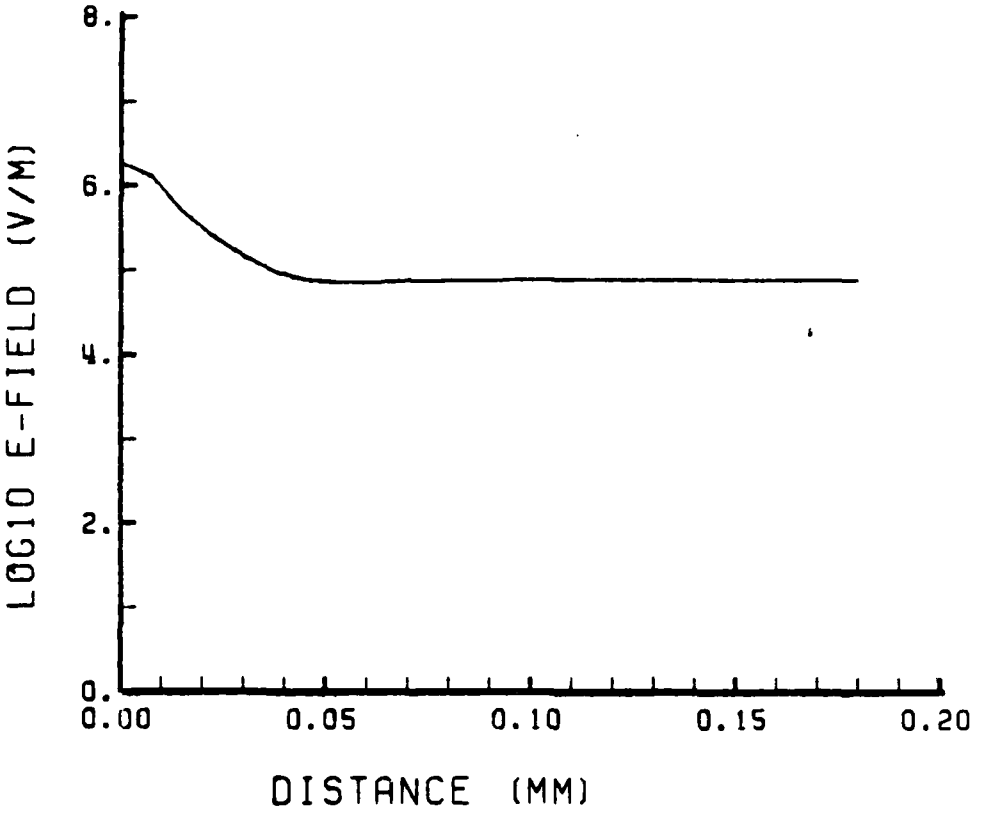


Figure 12-1

Presentation of the Electric Field (CASE I) along a line normal to the wall, extending from the active node site. (logarithm scaled)

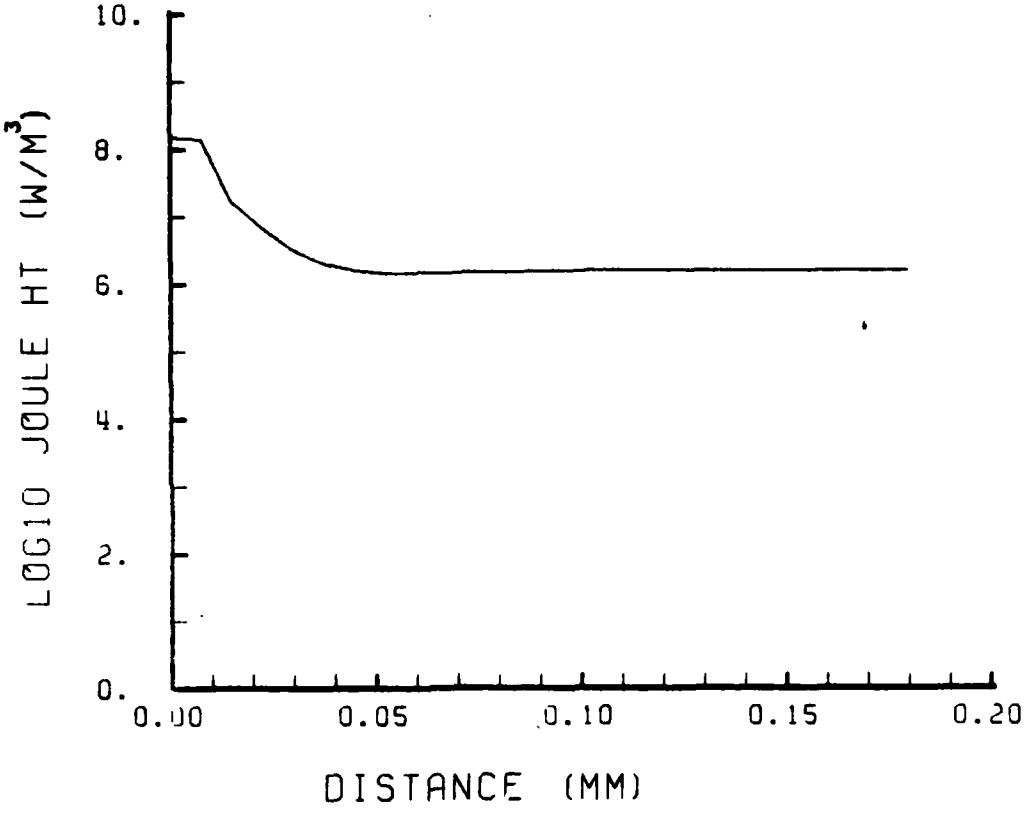


Figure 12-a

Presentation of the Joule Heating (CASE I) along a line normal to the wall, extending from the active node site. (logarithm scaled)

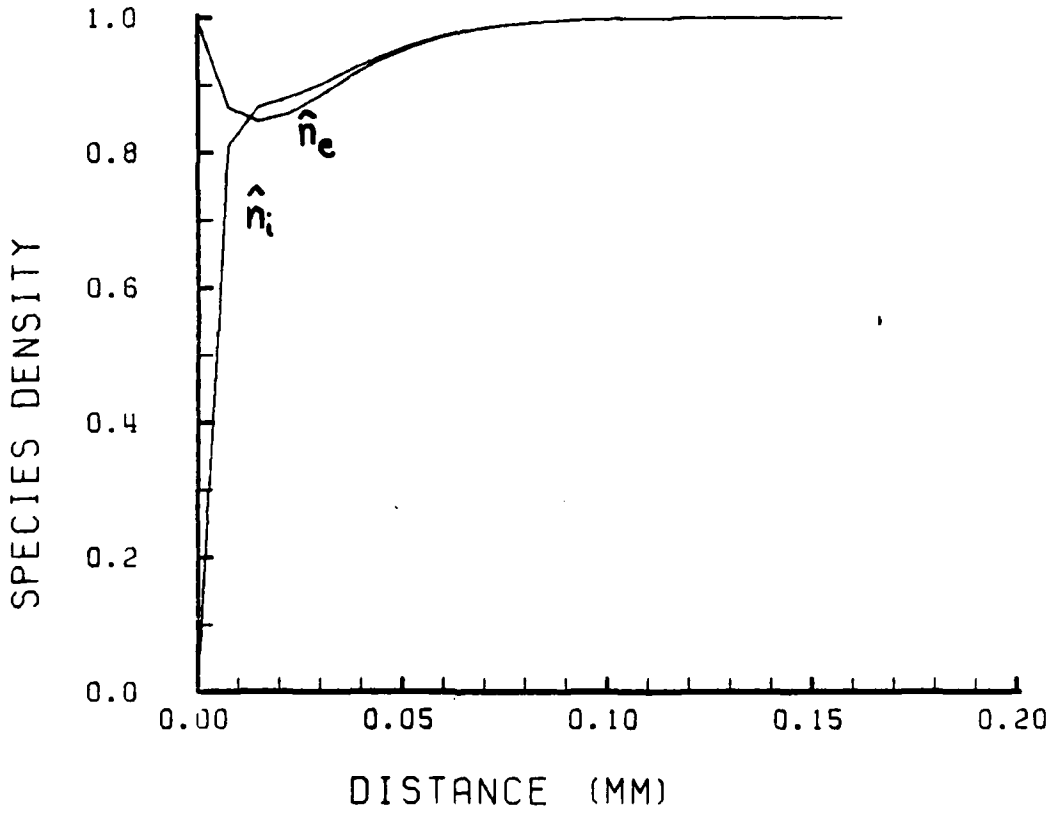


Figure 12-n

Presentation of the Species Densities (CASE I) on a line normal to the wall, extending along the inter-nodal symmetry line.

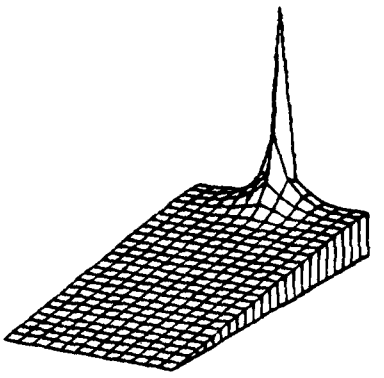


Figure 13-a

Oblique perspective of Potential Field (CASE II)

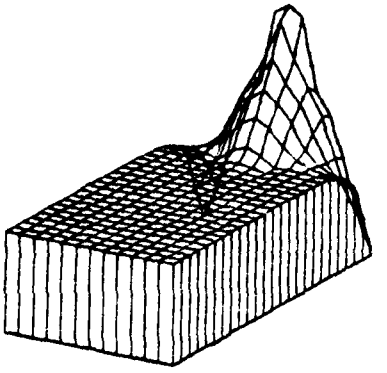


Figure 13-b

Oblique perspective of Temperature field (CASE II)

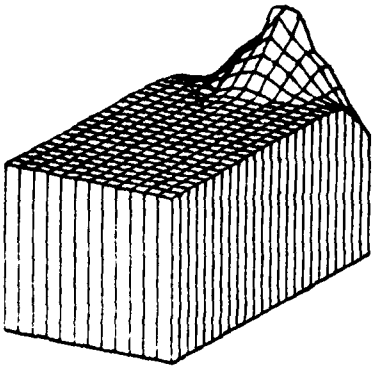


Figure 13-c

Oblique perspective of log Electric Field (CASE II)

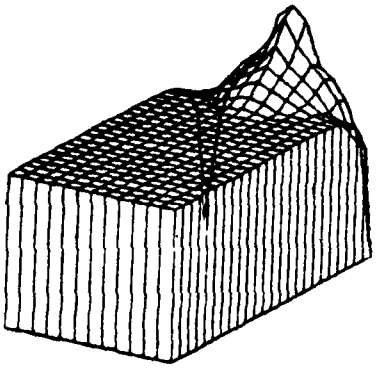


Figure 13-d

Oblique perspective of log Joule Heat (CASE II)

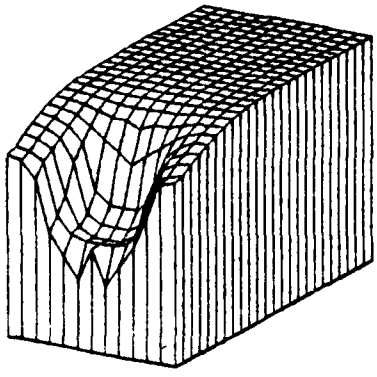


Figure 13-e

Oblique perspective of Electron Density (CASE II) (reversed perspective)

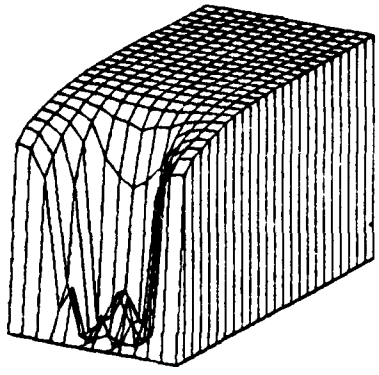


Figure 13-f

Oblique perspective of Ion Density (CASE II) (reversed perspective)

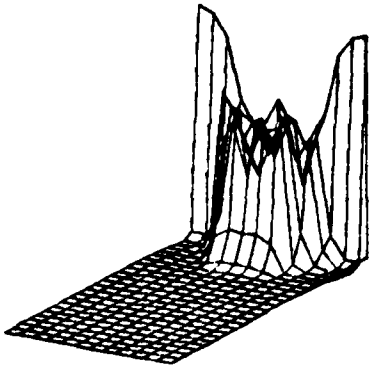


Figure 13-g

Oblique perspective of Space Charge (CASE II)

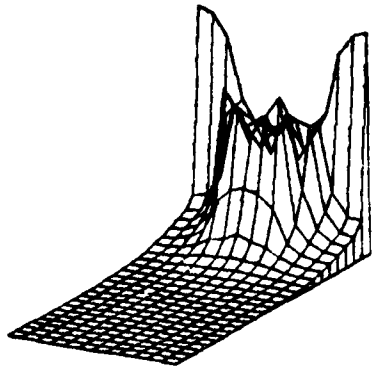


Figure 13-h

Oblique perspective of Net Production (CASE II)

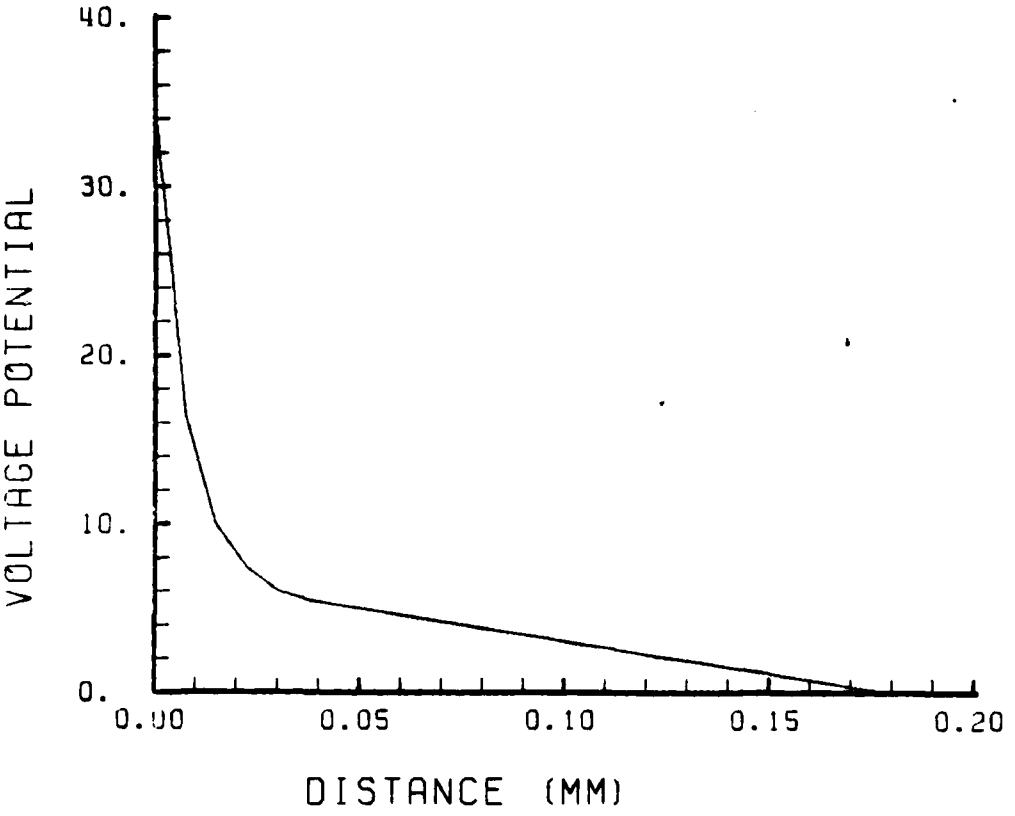


Figure 13-i

Presentation of the Voltage Potential (CASE II) along a line normal to the wall, extending from the active node site.

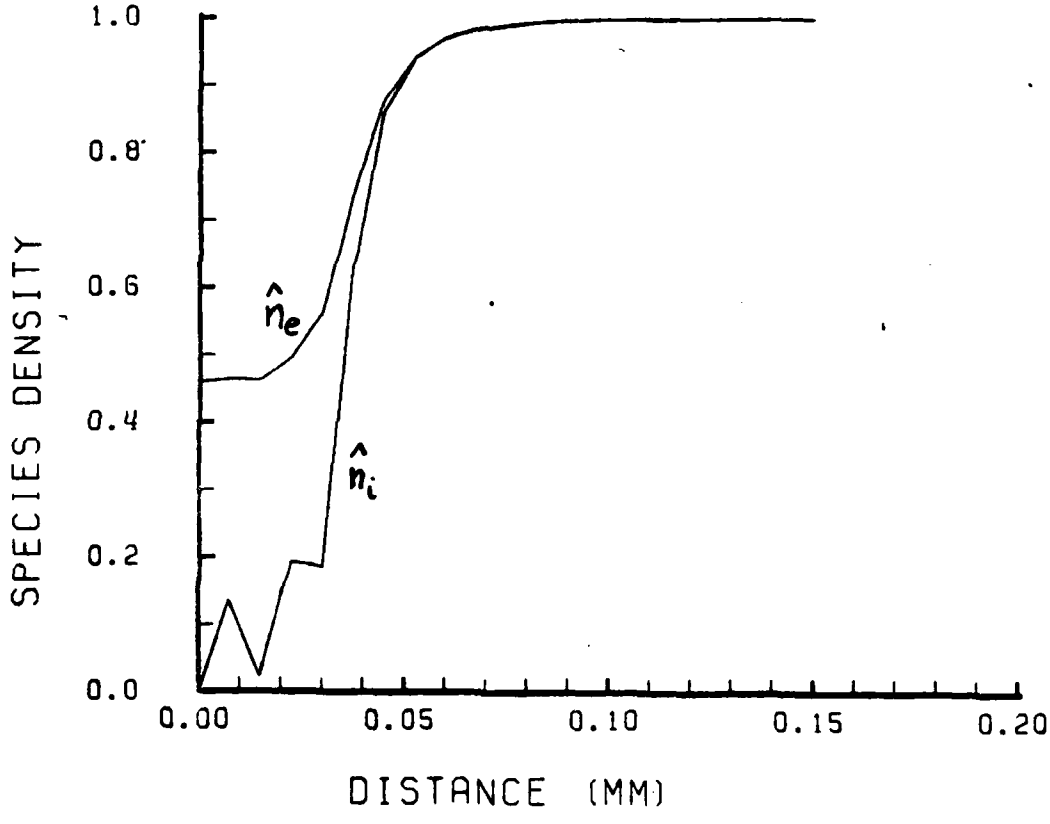


Figure 13-j

Presentation of Species Density (CASE II) along a line normal to the wall, extending from the active node site.

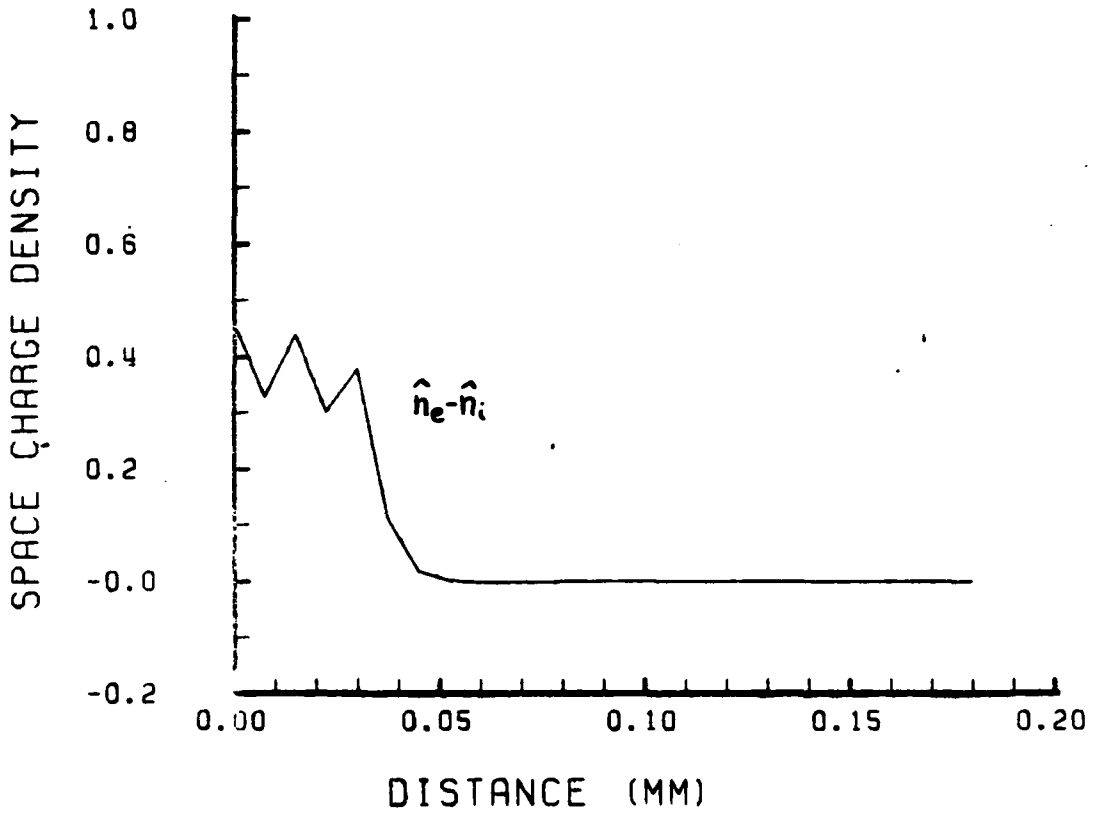


Figure 13-k

Presentation of the negative Space Charge Density (CASE II) along a line normal to the wall, extending from the active node site.

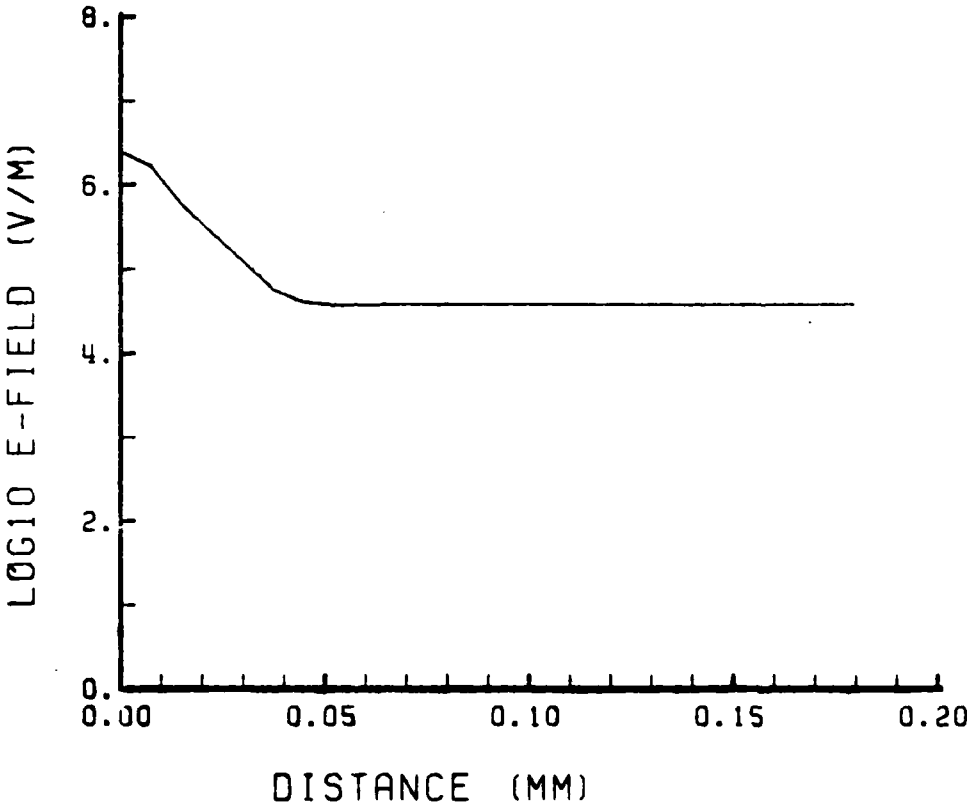


Figure 13-1

Presentation of the Electric Field (CASE II) along a line normal to the wall, extending from the active node site. (logarithm scaled)

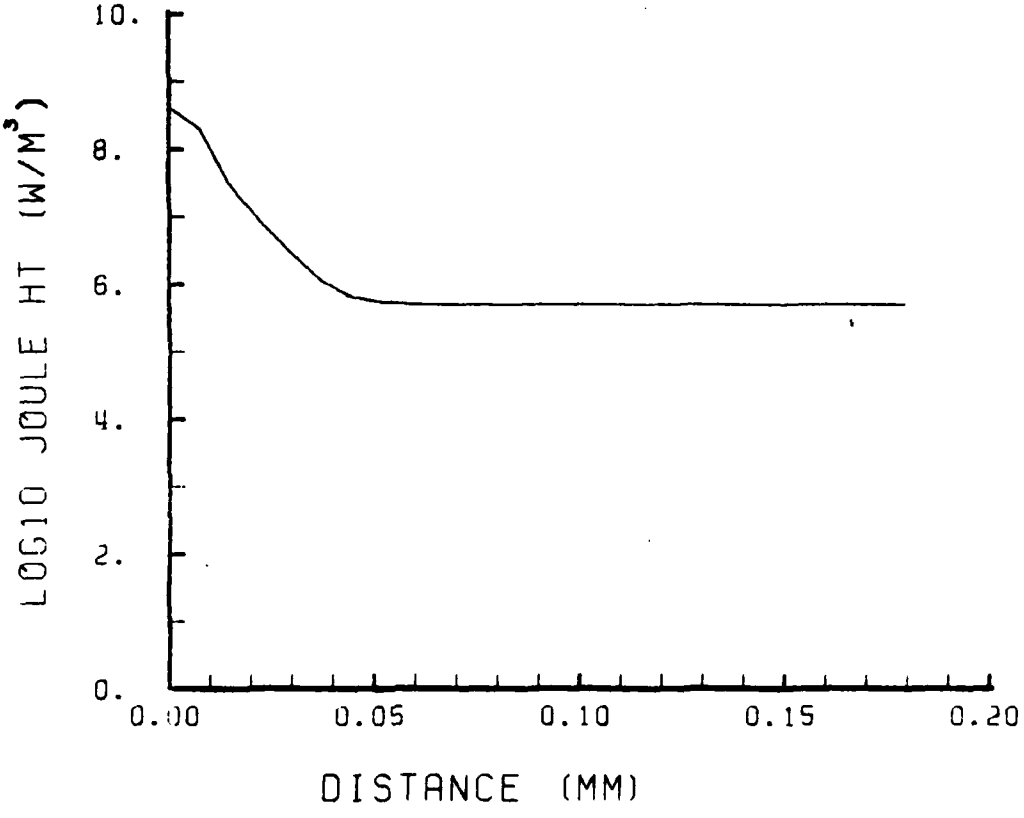


Figure 13-a

Presentation of the Joule Heating (CASE II) along a line normal to the wall, extending from the active node site. (logarithm scaled)

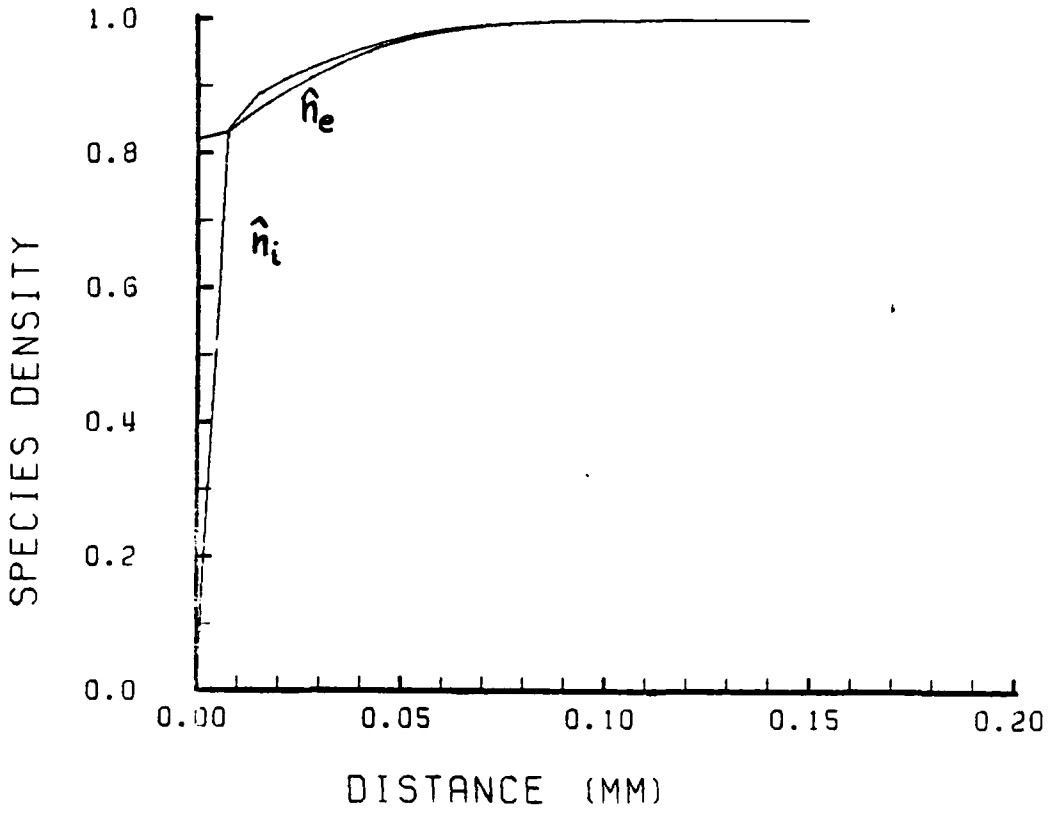


Figure 13-n

Presentation of the Species Densities (CASE II) on a line normal to the wall extending along the inter-nodal symmetry line.

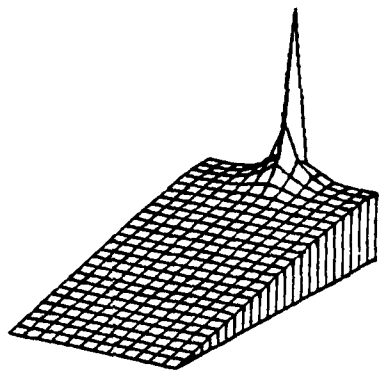


Figure 14-a

Oblique perspective of Potential Field (CASE III)

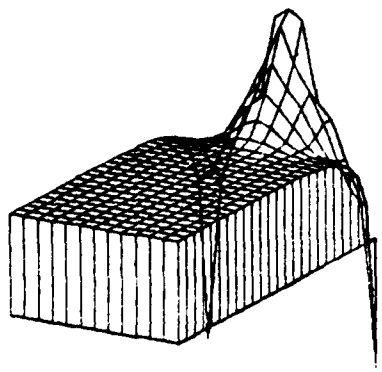


Figure 14-b

Oblique perspective of Temperature field (CASE III)

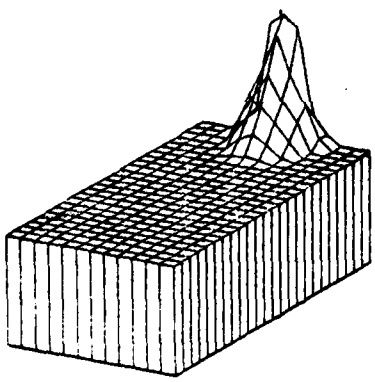


Figure 14-c

Oblique perspective of adjusted Temperature Field (CASE III)

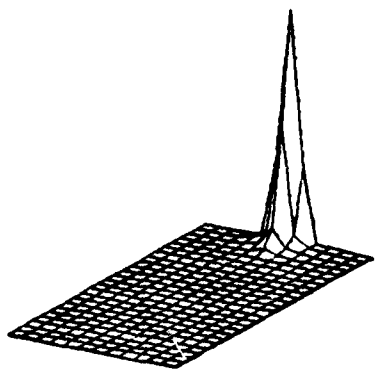


Figure 14-d

oblique perspective of Joule Heat (CASE III)

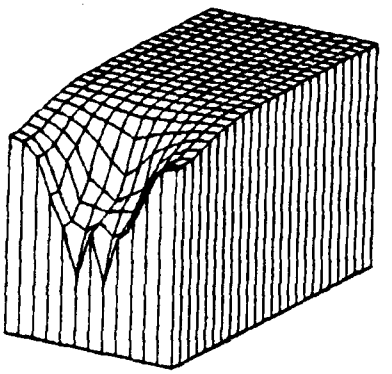


Figure 14-e

Oblique perspective of Electron Density (CASE III) (reversed perspective)

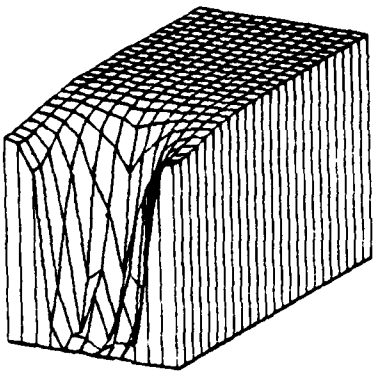


Figure 14-f

Oblique perspective of Ion Density (CASE III) (reversed perspective)

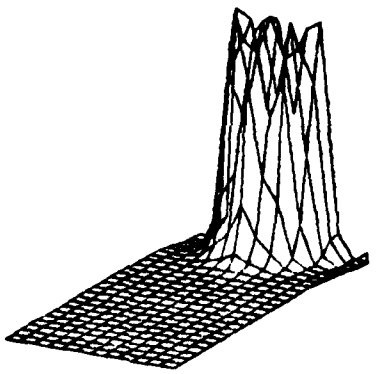


Figure 14-g

Oblique perspective of Space Charge (CASE III)

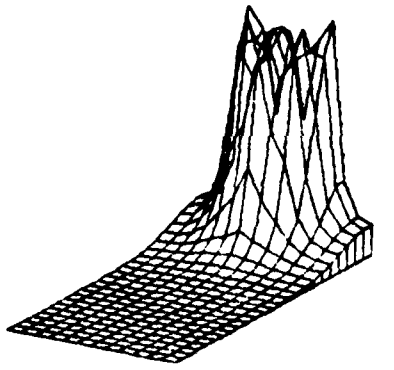


Figure 14-h

Oblique perspective of Net Production (CASE III)

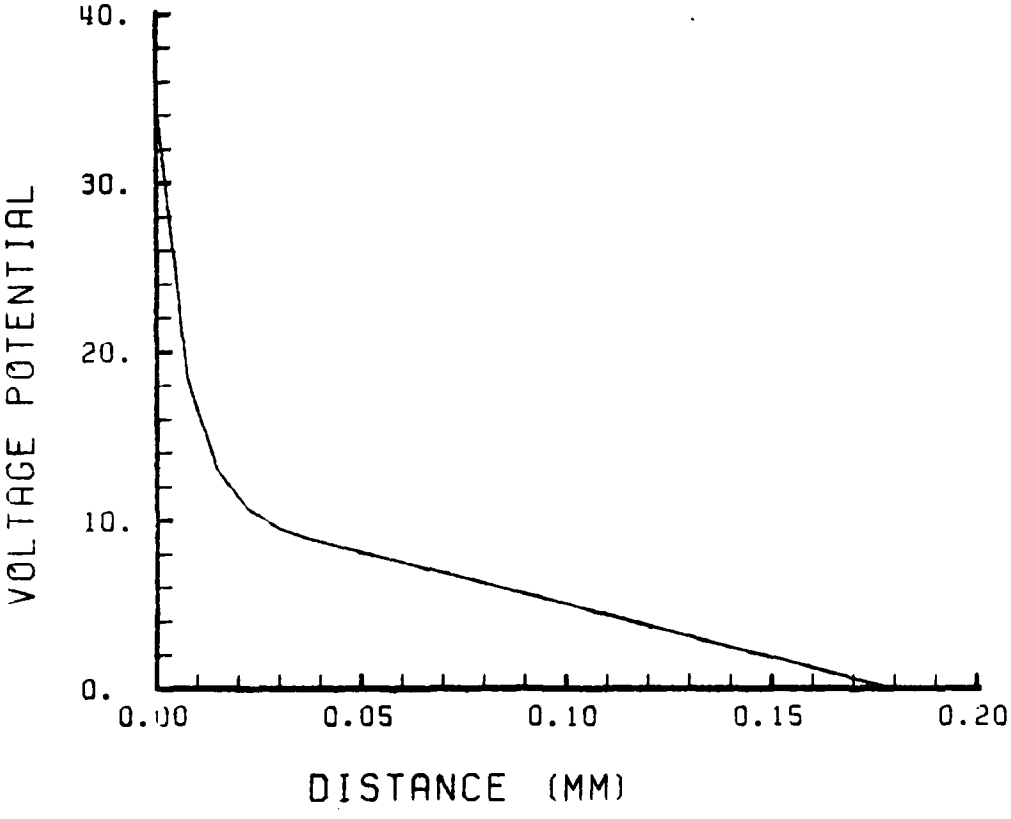


Figure 14-i

Presentation of the Voltage Potential (CASE III) along a line norm to the wall, extending from the active node site.

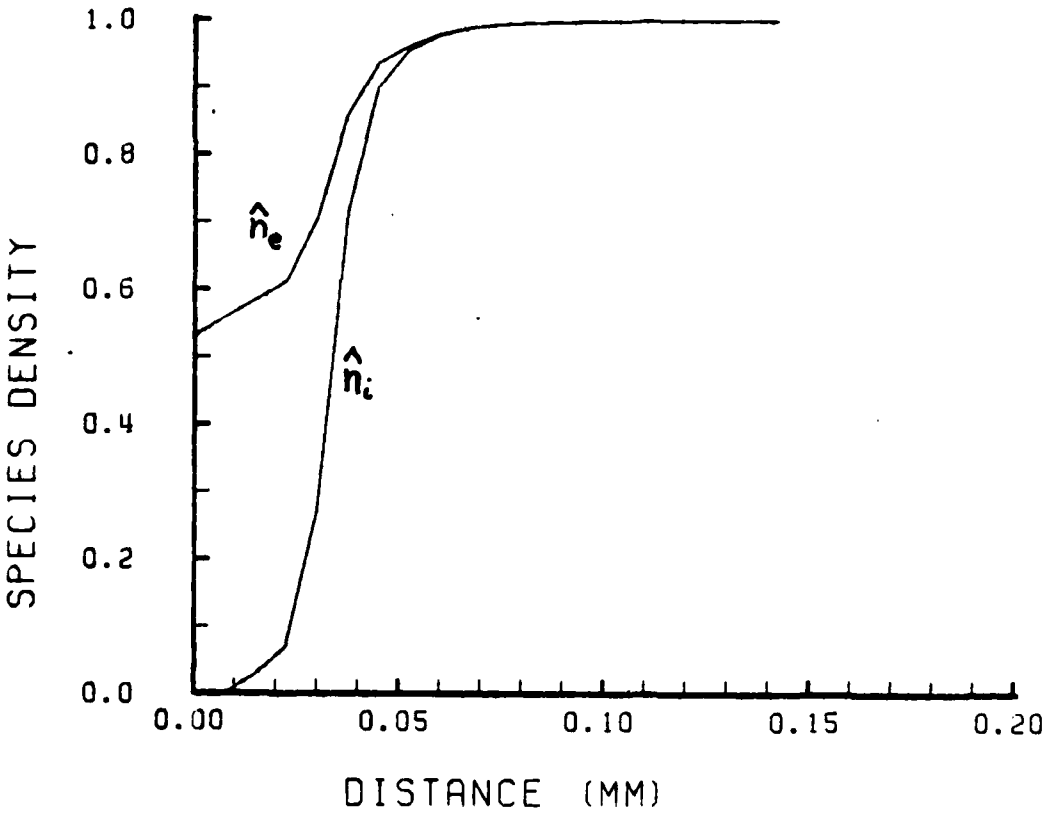


Figure 14-j

Presentation of Species Density (CASE III) along a line normal to the wall, extending from the active node site.

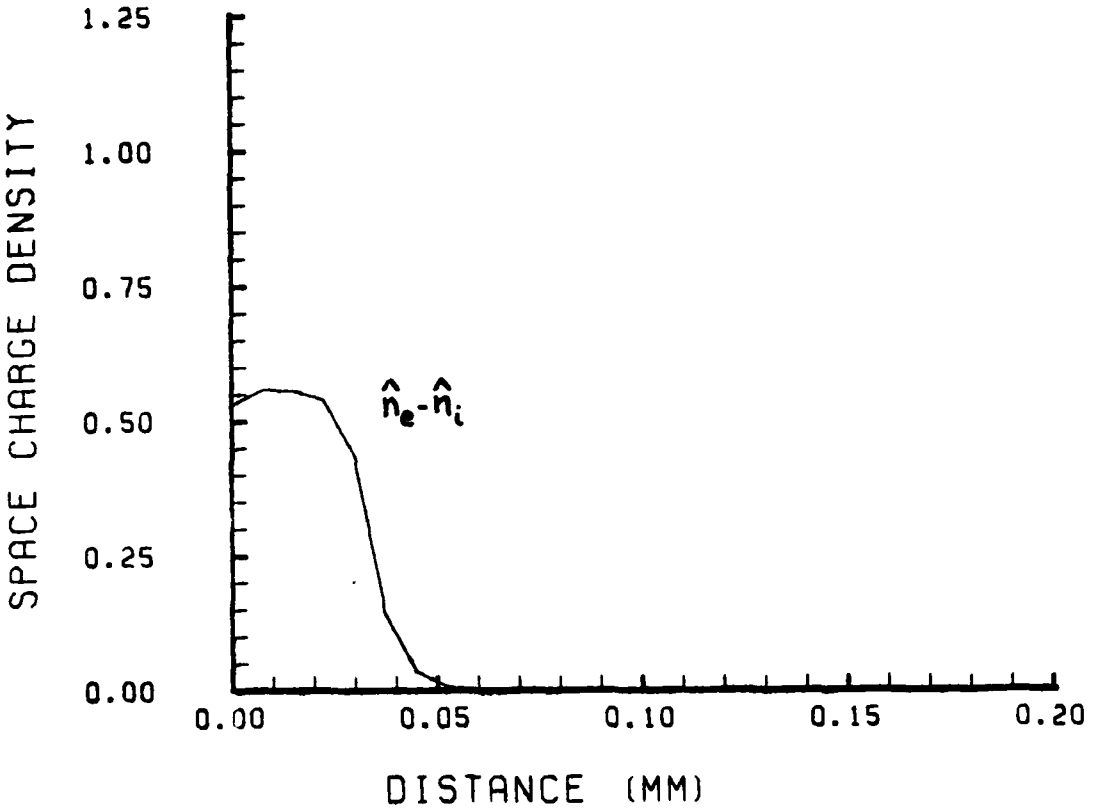


Figure 14-k

Presentation of the negative Space Charge Density (CASE III) along a line normal to the wall, extending from the active node site.

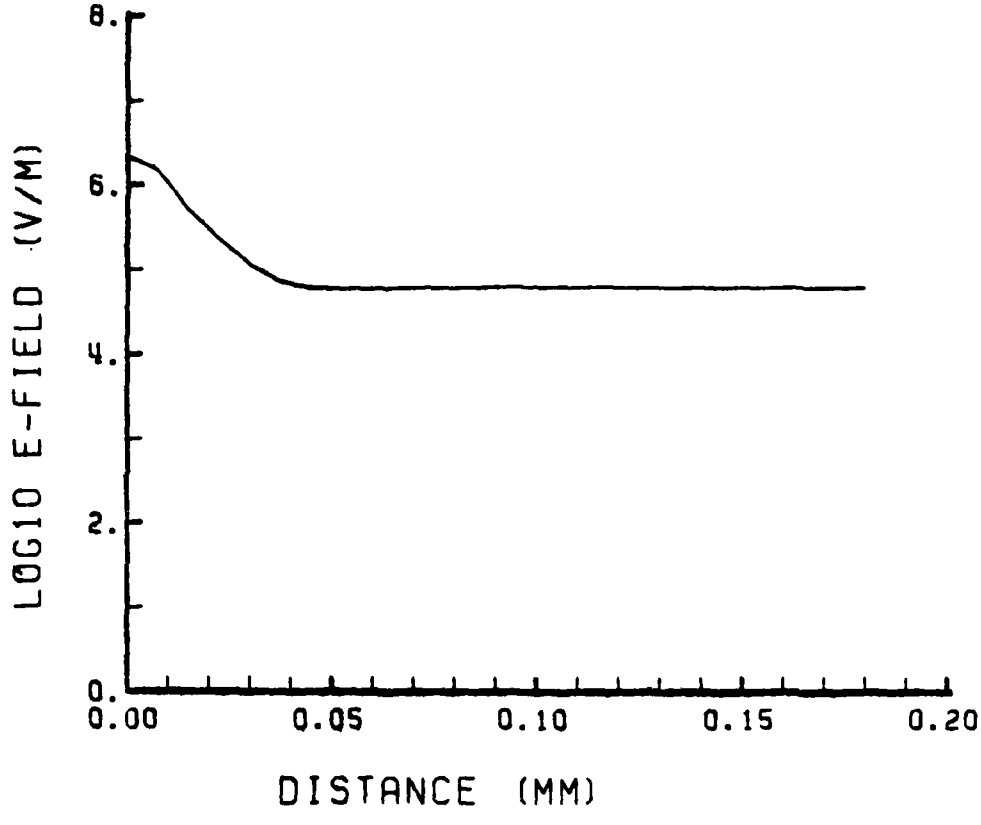


Figure 14-1

Presentation of the Electric Field (CASE III) along a line normal to the wall, extending from the active node site. (logarithm scaled)

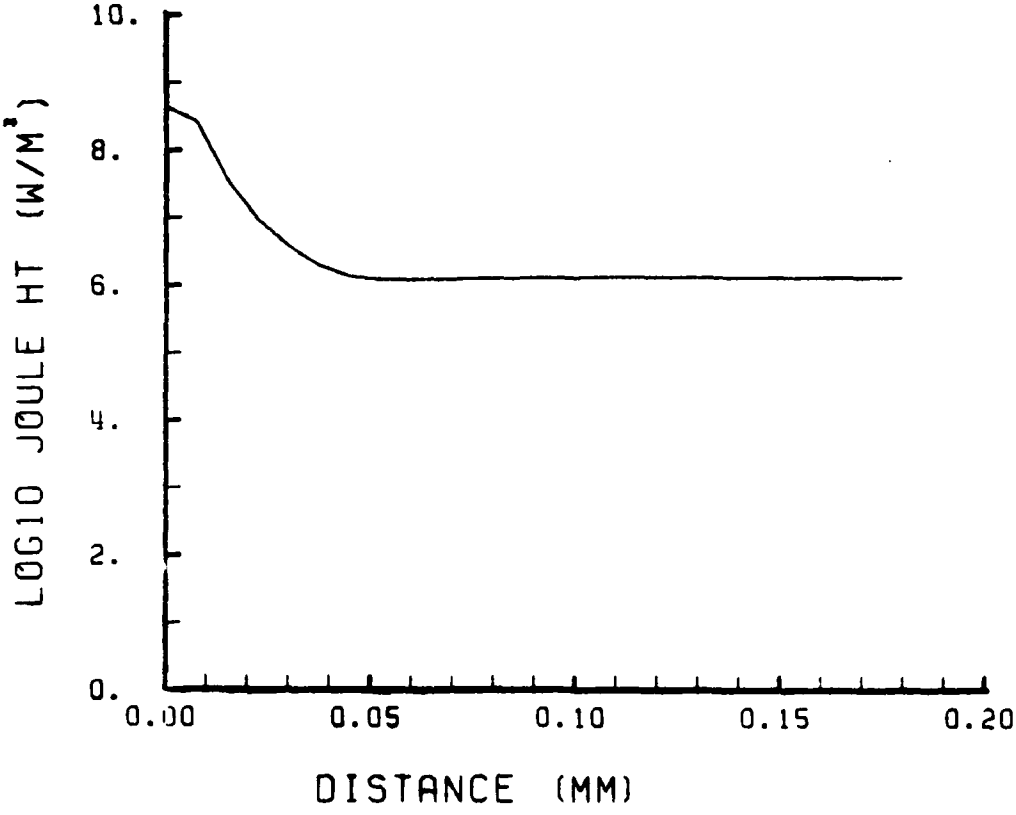


Figure 14-a

Presentation of the Joule Heating (CASE III) along a line normal to the wall, extending from the active node site. (logarithm scaled)

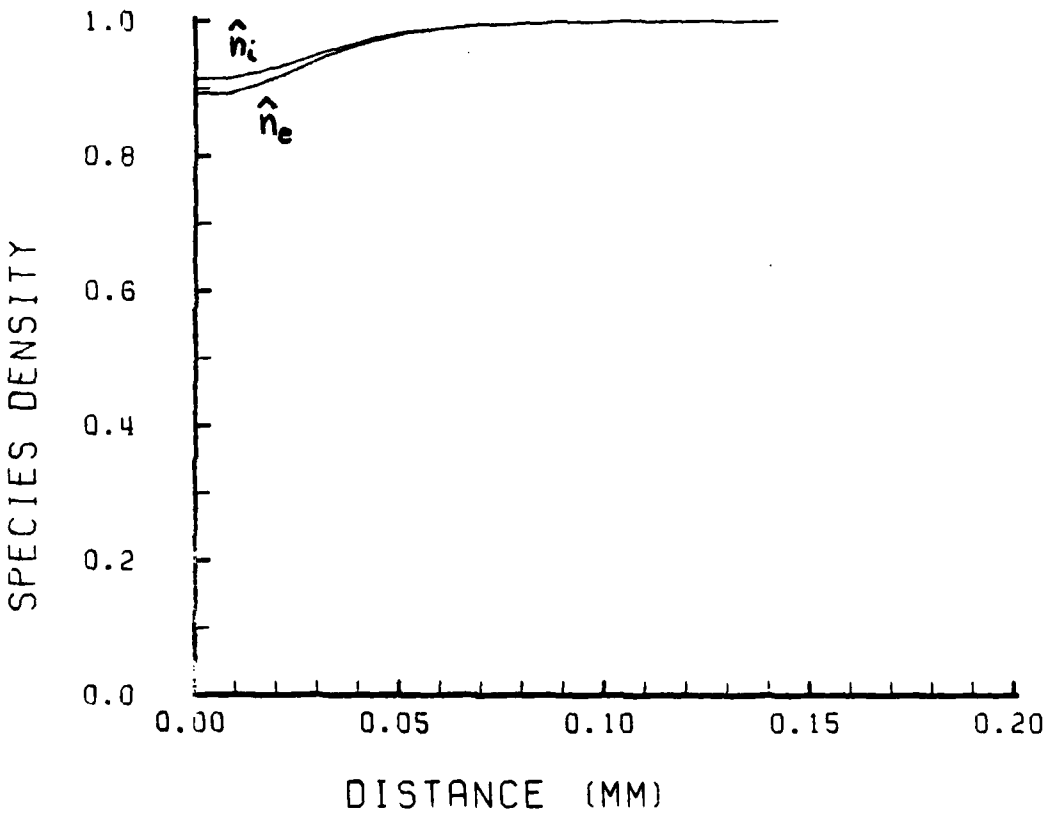


Figure 14-n

Presentation of the Species Densities (CASE III) on a line normal to the wall, extending along the inter-nodal symmetry line.

V. CONCLUSIONS AND RECOMMENDATIONS

A. REVIEW OF ASSUMPTIONS

In the development of this work several assumptions were made which the solution results later verified. The assumption of negligible convection effects is straight forward. The charged particle density boundary layer evolution time turns out to be small in relation to the pulse period; thus enabling consideration of the problem as a steady-state after-glow problem. The fact that the Joule heating of the gas is shown negligible, frees the formulation from an elaborate energy description. Since the electron temperature is a simple function of E/n, the problem can be further simplified. The solution proceeds with nominal values of the parameters of the discharge as discussed in Sec. II.B. Finally, the importance placed on the net production term is verified by the profound changes in the results including the fact that "$\dot{n}_e$" is necessary for the establishment of the charged-species boundary-layer behaviour. The analysis of App. B shows that an assumption of "frozen flow" would in fact be premature.

B. PHYSICAL CONCLUSIONS

The present model of the sheath and ambipolar region evolves from the assumptions of steady state, uniform ion temperature and constant diffusion coefficients. A summary of important results is presented in Table II.

TABLE II: SUMMARY OF IMPORTANT RESULTS

SUMMARIZED RESULTS	CASE I	CASE II	CASE III	(units)
sheath length	3.8	3.5	3.7	10^{-5} m
ambipolar length	4.5	3.5	3.7	10^{-5} m
E_{∞} (free-stream)	7.9	3.9	6.3	10^{4} V/m
E_{o} (anode)	1.9	2.5	2.2	10^{6} V/m

The results show that a sheath develops as anticipated from theory and the proposed assumptions. The sheath characteristic length is consistent with that predicted by Debye shielding. Debye shielding of a flat plate predicts that the sheath length "λ_{s}" is 4.4×10^{-5} meters. For a point (or spherical) shielding the sheath length is 3.1×10^{-5} meters. The present study involves the shielding of a point voltage source situated on a surface charged plate. Since both two- and three-dimensional effects are inherent in the formulation, the resulting sheath lengths are quite reasonable.

As Table II shows, the size of the ambipolar region is of the same order as the sheath size. Here the ambipolar length is defined as the length extending from the edge of the sheath to the point at which 99% of the free-stream density is recovered. Finally, the sheath can be self-generating from a consistent set of equations without an overall energy equation.

The resulting electric fields at the free-stream boundary do not quite attain the desired 2.7×10^5 V/m presented in the problem formulation. These results fall short by the same factors required for the adjustment of the electron temperature field to meet the stated free-stream boundary requirements. It must be pointed out that the electric field is the required boundary condition. However, this boundary condition could be made to yield no reasonable results, and so the chosen boundary condition is $\phi = 0$.

The resulting electric fields at the active site in each of Cases I, II, or III was of two orders of magnitude higher than that of the free-stream. These electric fields are below that needed for breakdown of Nitrogen at one atmosphere, which is determined as, [Ref. 18];

$$E_b = (E/p)p = (350 \text{ V/cm-torr}) \times (760 \text{ torr}) = 2.7 \times 10^7 \text{ V/m}$$

The requirement of strictly matched currents produces steep derivatives and inflections of the density profiles in the vicinity of the anode. A noticeable "spike" is evident in Fig. 12-j for example. Case II and III anode electron densities were established on more aesthetic grounds, where smoother appearing density profiles were generated. In these cases the anode current exceeded the free stream current by only a factor of 3.

The nonactive regions of the electrode (wall) far from the active site reveal a small positive space charge density amounting to approximately 2% of the free-stream density. The negative space charged sheath is 46% to 56% of the free-stream density.

Previous work [Ref. 12] of a similar problem reported no reasonable results for the case of a catalytic wall ($n_e = n_i = 0$). The results presented in this study demonstrate wall conditions of $n_i = 0.$ and $n_e < 1$. Here, a noticeable positive space charge coexists with the large negative space charged sheath, without large inflections in either the voltage potential or densities.

The Case III study has both electron and ion wall conditions floated. The shape of these density distributions is quite symmetrical about the anode site in all directions.

One needs little imagination to visualize that the density profiles can be "wrapped" around the anode completely, in effect treating the anode as a whisker or protrusion into the plasma. However, the imagination must be tempered by the fact that it can only be a two-dimensional protrusion (e.g., a "blade").

The effect of the net production term has been previously discussed as one of the most important results of this study. The term is necessary for inclusion in the problem in order (1) to attain wall electron density values of less than the free-stream, (2) to produce the boundary-layer behavior of the species densities,and (3) to obtain converged solutions with voltages greater than 10 volts or computational node spacings of any order less than that of the sheath length.

C. NUMERICAL CONCLUSIONS

The Modified Newton-Raphson (MNR) method, presented to solve the particular set of coupled non-linear partial differential equations in this study, is a general method that can be applied to more sophisticated systems. The deficiency of the quasi-Jacobian method (discussed in Sec. III.C and App. C.) in dealing with non-linear terms is circumvented by an alternative linearizing scheme. Convergence is

rapid; however, the starting solution must not exceed 10 volts, must have zero net production, and must have a constant electron temperature during the first 2-3 iterations. After the solutions take their initial form at ten volts, the potential field is "amplified" and reinstated as a new starting solution. The final stages of the converging solution are carried out under the full influences of (1) floating wall densities, (2) variable electron temperature, (3) net production and (4) an active node site electron density being solved for through current matching.

The profiles of the space charge and net production distributions demonstrate erratic behavior at the first few computational nodes from the active site. This is believed to be caused by the linearization of the governing non-linear equations and has the appearance of truncation error when observed on an iteration to iteration basis. However, the remaining regions of the profiles are smooth.

D. RECOMMENDATIONS

This work succeeded in solving the problem in a two-dimensional environment. This fact necessarily limits the quantitative results since the current density decreases in only two dimensions. With the computational power of recent computers, it should prove worthwhile to make the next

advance as a three-dimensional problem rather than elaborate in the two-dimensional case. A three-dimensional scheme would render more realistic results of a quantitative nature because active spots are more likely to exist than active lines and because 3-D geometries require less voltage for a given current.

Specific recommendations other than the above include;

- Account for the ion contribution to the current boundary condition into the wall.
- Investigate the physical implications of the low electron temperature near the wall far from the anode site.
- Investigate the physical implications of the electron density "spikes" prevalent near the anode site.
- Make the wall temperature a valid boundary condition.
- Use a free-stream boundary condition of $E_\infty = 2.7 \times 10^5$ V/m, rather than zero potential. This would introduce desired results for pumping lasers.
- Include a more sophisticated electron temperature model, or verify the present one.
- Increase the computational mesh density.
- Study the general application limits of the MNR method.
- Investigate "n " in the role of making one-dimensional solutions possible (i.e., alter the reacting rates).
- Investigate whether the low electron temperatures and associated steep derivatives located along the wall between active sites have any physical implications, or result merely from inadequate problem formulation.
- Consider the electron-beam ionization in the solution to produce more realistic effects.

APPENDIX A

CHARACTERISTIC PARAMETERS

A. SHEATH LENGTH

The extent of the sheath is one of the most important charcteristic lengths in this study. It is within the sheath that most of the potential drop occurs for a short discharge. Fortunately, the sheath length can be estimated rather easily from Poisson's equation, (Eq. A1)

$$\nabla^2\phi = \frac{e}{\epsilon_0}(n_e - n_i) \tag{A1}$$

Let;

$$\hat{\phi} = \frac{e|\phi_0 - \phi_\infty|}{kT_{e\infty}} = \frac{e|\phi|}{kT_{e\infty}} \quad ; \quad \hat{n}_s = n_s/n_{e\infty} \tag{A2}$$

where "λ_s" is the characteristic sheath length; the subscript "o" indicates the value at the electrode and "∞" the value at the undisturbed plasma. After gathering the dimensional quantities on one side, Eq. A2 becomes;

$$\hat{\nabla}^2\hat{\phi} = \frac{e\,n_{e\infty}\lambda_s^2}{\epsilon_0|\phi|}\cdot\left[\hat{n}_e - \hat{n}_i\right] \tag{A3}$$

The characteristic sheath length "λ_s", defined in this fashion, is used to non-dimensionalize the "del" operator in all directions. That is, independent of the dimensionality

of the problem, the sheath is effective over the characteristic length just defined. Now an estimate of the size of the sheath can made by setting the grouping of dimensional quantities in Eq. A3 to be of the order one, namely;

$$\lambda = \left[\frac{\epsilon_0 |\phi|}{e n_{e\infty}}\right]^{1/2} = \lambda_{D_\infty}\left[\frac{e|\phi|}{kT_{e\infty}}\right]^{1/2} \qquad (A4)$$

where "λ_{D_∞}" is the Debye length in the undisturbed plasma.

Anticipated values are now established for the sheath length. Take for example a 35 volt anode potential, and a charge species density of 10^{17} to 10^{19} m^{-3}. These parameters define a sheath length of 1.4×10^{-4} to 1.4×10^{-5} meters.

As can be seen, a ten-fold increase in voltage would change "λ_s" by a factor of 3, rendering the nondimensionalization of the problem relatively insensitive to the chosen value of the voltage potential. That is to say the value of the sheath characteristic length as defined may be considered a resonable estimate for the charge species densities examplified.

The important implication here is that extreme care must be devoted to electrode surface preparation to preclude whiskers or flaws of a size comparable to "λ_s". These protrusions would produce a focusing of the electric potential to create extremely high electric fields, which become sites

for initiation of breakdown. Protrusion growth has been observed to form in a time on the order of milliseconds. Many conditions are favorable to protrusion growth, dependent on the pre-breakdown and breakdown current profile, electrode composition (especially in the case of a thermionic cathode), surface purity and surface preparation [Ref. 21]. Some possibilities are;

- Electrode plastic flow or melting in an electric field due to micro-particle bombardment.
- Sticking of microparticles.
- Electron beam induced protrusion growth.

B. DIFFUSION LENGTH

If diffusion is a significant factor in this study then its contribution to the current would be;

$$J_x = D_e e\frac{\partial n_e}{\partial x} - D_i e\frac{\partial n_e}{\partial x}$$

or, since $D_e > D_i$, and using Einstein's relation, or $D/\mu_e = kT_e/e$;

$$J \approx eD_e\frac{\partial n_e}{\partial x} = \mu_e kT_e\frac{\partial n_e}{\partial x}$$

making the equation unit free (through fractional analysis);

$$\frac{JL}{eD_e n_{e_0}} = \frac{\partial n_e}{\partial x} \sim \text{ORDER}[1] \qquad (A5)$$

If the right side of Eq. A5 is of the order one, (the nondimensional density gradient is a maximum of 1) then "L" may be estimated as;

$$L \approx eD_e n_{e_\infty}/J \approx 3.7\times10^{-6} \text{ m}$$

for the values;

$n_e = 10^{18}$ m^{-3}

$D_e = 0.0665$ m^2/(V•sec)

$J_e = \mu_e \bullet n_e \bullet \nabla\phi = \mu_e \bullet e \bullet n_e \bullet n_\infty \bullet (E/n) \approx 2.88\times10^3$ A-m^{-3}

Since $\lambda_s > L$, then any role attributed to diffusion should be visible within the sheath. The E/n within the thin sheath is of course greater than the E/n at the free stream (10^{20} V•m^2), which decreases the local value for "L" still further.

Another approach is to assume a one-dimensional solution to the charged species conservation equations, which can only be done outside of the sheath, i.e., in the ambipolar region. The ion conservation equation may be written;

$$\dot{n} = \nabla \bullet J = e\nabla \bullet [\nabla n \nabla\phi - D\nabla n] \qquad (A6)$$

Since it can be shown (next section) that the production term can be approximated as;

$$\dot{n} \approx \nu_i n - \alpha n^2$$

then for the stationary condition of no net production;

$$\nu_i = \alpha N$$

Nondimensionalizing Eq. A6, it can be shown that the

characteristic diffusion length "L" is;

$$L = \sqrt{D/\alpha N} = 2.\times10^{-6} \text{ meters}$$

which is of the same order of magnitude as the characteristic sheath length.

C. IONIZATION/RECOMBINATION LENGTH

Analysis of the production term (Eq. 4) reveals that for Nitrogen the three-body (electron) collisional recombination rate (α_2) is small in relation to to the other terms, (see App B.). In any case the net production term is;

$$\dot{n}_e = \nu_i n_e - \alpha_1 n_e n_i - \alpha_3 n_e n_i n_e \qquad (A7)$$

In the steady state the net production term is zero, and as is done in Appendix B, the relation becomes;

$$\dot{n}_e = (\alpha_1 + \alpha_3 n_e) \cdot (n^* - n_e) \cdot n_e$$

substituting the value of 0.17×10^{-11} m^3/sec, and $(n^* - n_e) \approx 10^{18}$, we can estimate the characteristic production time to be $\tau_p = 5.88\times10^{-7}$ sec.

To establish a meaningful characteristic length, the average thermal velocity will be used since it represents the principal transport velocity of the medium.

$$\ell_p = \tau_p \cdot \bar{v}_e = 5.88\times10^{-7} \cdot 3.2\times10^{2} = 1.88\times10^{-4} \text{ m}$$

The characteristic length for net production is a factor of four larger than the sheath length ($\lambda_s = 4.4\times10^{-5}$ m). The

conclusion which can be made from this (fractional) analysis is that the net production is significant within the overall region of interest, including the sheath itself.

D. FROZEN FLOW

When a process relaxation time "τ_r" is long compared with the time that a fluid particle spends in a given region, the flow is said to be "frozen" in this region. By this term one means that a reaction process may be neglected, as far as influencing the flow is concerned, and the species involved treated as chemically inert, [Ref. 22]. In the opposite limit, where "τ_r" is very small relative to "τ_f" the characteristic flow time, the flow is said to be in "equilibrium." For the recombination rates given in this section, "τ_r" = $6.\times10^{-7}$ and "τ_f" = 4.0×10^{-7} seconds. Here flow time is defined as the sheath characteristic length divided by the drift velocty. Thus the flow is not "frozen" as far as chemical reaction, but we must consider that the electron and ion species are not in thermal equlibrium.

APPENDIX B

DEVELOPMENT OF THE PRODUCTION TERM

The net production term is defined as the difference between ionization and recombination rates. Whenever a neutral atom or molecule loses one or more of its electrons, ionization has ocurred. Recombination is the process wherein positive ions and electrons combine to form neutral atoms or molecules. It is simply reverse ionization. If the net production is zero then ionization and recombination are equal. The net production term is the right side of Eqs. 2 and 3, the expression for which is developed here.

This work considers only three types of <u>recombination</u>

(1) <u>Dissociative</u> - the electron recombines with a molecular ion $(AB)^+$, and the recombination energy goes into dissociating the molecule and increasing the kinetic energy of the resulting products. The process is described as;

$$e + (AB)^+ \longrightarrow A + B$$

(2) <u>Three-body (electron)</u> - the energy released in the recombination process is carried off by a third body which is an electron. The process is described as;

$$e + A^+ + e \longrightarrow A + e$$

(3) Three-body (heavy) - the recombination energy is carried off by a third body which is a heavy particle such as an atom or molecule. The process is described as;

$$e + A^+ + B \longrightarrow A + B$$

This study does not consider other recombination mechanisms such as radiative or dielectric recombination, since they are generally of low significance in relation to the just described processes. The ions are limited to singly positive ions. No account is made for intermediate or excited states as this requires a detailed balancing which is beyond the scope of this work. Additionally, no account is made of attachment.

Rewriting Eq. 4 for simplicity of discussion;

$$\dot{n}_e = \nu_i n_e - \alpha_1 n_e n_i - \alpha_2 n_e^2 n_i - \alpha_3 n_e n_i n_o \qquad (B1)$$

where each "α_i" refers to the recombination coefficients for dissociation, three-body (electron) and three-body (heavy), respectively.

The recombination coefficients depend upon the ionic species involved. Some general but representative values [Ref. 22] follow;

$$\alpha_1 = 10^{-13}\ m^3/sec$$
$$\alpha_2 = 9 \times 10^{-14}\ m^3/sec$$
$$\alpha_3 = 2 \times 10^{-13}\ m^3/sec \text{ (air)}$$

Hinnov and Hirschberg [Ref. 23] developed a relation for three-body (electron) recombination coefficients as a function of electron temperature (T_e) and density, valid for energies less than 0.25 eV.

$$\alpha_2 = 1.09\times10^{-20} n_e T_e^{-9/2} \text{ m}^3/\text{sec}$$

where the electron temperature is °K. Gurevich and Pitaenski [Ref. 24] extended this development to obtain;

$$\alpha_2 = 2.13\times10^{-20} n_e T_e^{-9/5} \ln\Lambda \text{ m}^3/\text{sec}$$

where;

$$\ln\Lambda = 1.24\times10^{7} \ (T_e^3/n_e)^{.5}$$

At 1.0 eV these coefficients are 5.6×10^{-21} and 1.05×10^{-10} m^3/sec respectively. A simplification of the later expression for α_2 is valid from energies of 0.1 to 10.0 eV and $n_e = 10^{-18}$ m^{-3};

$$\alpha_2 = 0.032 \times T^{-4.3}$$

Brown [Ref. 18: p.197] gives a value for the dissociative recombination coefficient;

$$\alpha_1 = 1.5\times10^{-12} \text{ m}^3/\text{s}$$

The ionization coefficient can be determined from considering that there is no net production in the free stream, i.e., far from the electrode. Here;

$$0 = \nu_i N - \alpha_1 N^2 - \alpha_2 N^3 - \alpha_3 N^2 n_e \qquad \text{(B2)}$$

where "N" is the stationary state. Solving for and substituting into Eq B1 gives;

$$\dot{n}_e = n_e(N-n_i)(\alpha_1+\alpha_3 n_e) + \alpha_2 n_e T_e^{9/2}(N^2-n_e n_i) \qquad (B3)$$

where;

$$\alpha_1 = 1.5\times10^{-12} \qquad \text{(from Ref. 18)}$$

$$\alpha_2 n_e = 1.09\times10^{-20} \qquad \text{(from Ref. 23)}$$

$$\alpha_3 n_e = 2.0\times10^{-13} \qquad \text{(from Ref. 22)}$$

The net production term is the relation, suitably non-dimensionalized, that appears in the computer subprogram "RTSIDE."

APPENDIX C

NUMERICAL SOLUTION METHOD

A. GENERAL DISCUSSION

There are two basic approaches used in solving a nonlinear system of equations, such as possed in this study. A given method can usually be classified as one of the ascent (or descent) or more commonly refered to as gradient methods, or as one of the Newton methods. The method applied in this study and by Dolson [Ref. 12] is one of the Newton methods. A brief review of both methods will be presented to facilitate development of the particular method employed in this study.

Let the nonlinear system of equations be written as;

$$\begin{aligned} f_1(x_1, x_2, \ldots, x_n) &= 0 \\ &\cdots \\ f_n(x_1, x_2, \ldots, x_n) &= 0 \end{aligned} \tag{C1}$$

or in vector form for convenience;

$$f(\vec{x}) = 0 \tag{C2}$$

In the descent method the problem is possed in such a way so as to form a function which is zero at any solution and positive otherwise. A typical such function would be;

$$\phi(x_1, x_2, \ldots x_n) = \sum_{i=1}^{n} |f_i|^2 \qquad \text{(C3)}$$

so that the problem is reduced to finding a minimum of ϕ, which also happens to be the value zero. In solving problems by this method, an initial guess to the solution must be provided, call it $x^{(1)}$. Then a direction d is determined so as to point to a better solution, $x^{(2)}$. This direction is often chosen to be the gradient of $\phi(x)$. The operation -grad $\phi(x)$ is the direction in which $\phi(x)$ decreases most, and is locally the best direction in which to reduce $\phi(x)$.

$$d = -\text{grad } \phi(x)$$

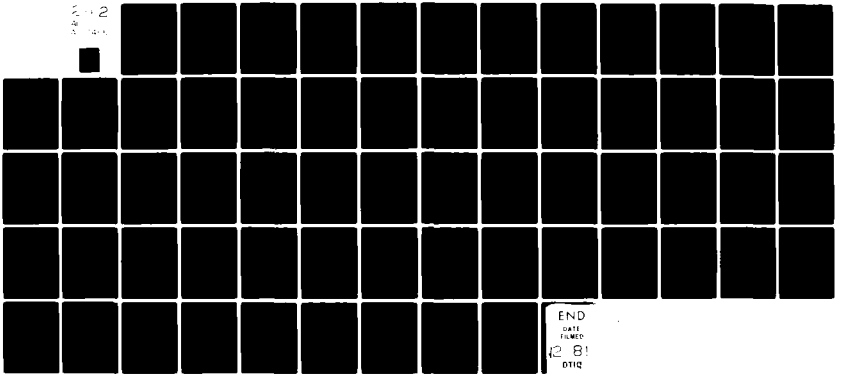
so that;

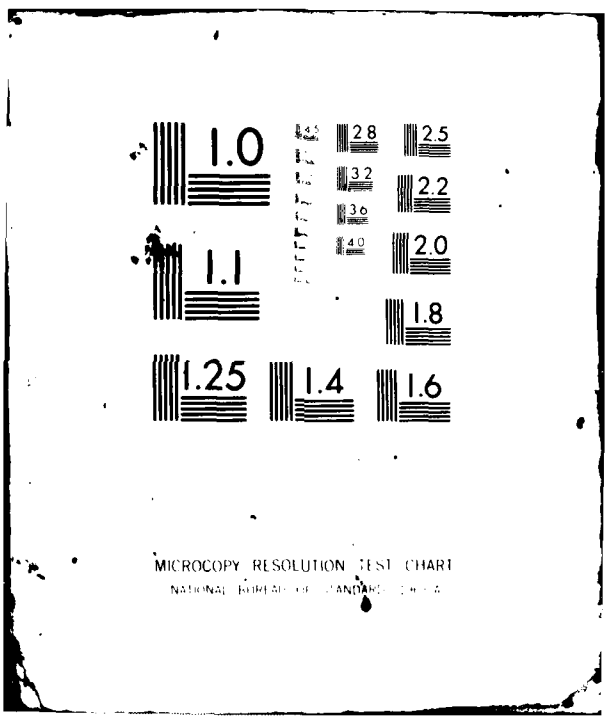
$$x^{(2)} = x^{(1)} - a \cdot \text{grad } \phi(x^{(1)})$$

where "a" is chosen to be an appropriate step size. A problem is encountered at local minima of $\phi(x)$, which needs to be avoided. One must also use the step "a" in such a manner as to control the acceleration along slowly or rapidly varying $\phi(x)$. It may be that by the nature of $\phi(x)$, the gradient may have to be approximated by finite differences, an approach which offers a flexible recourse even when not required.

AD-A107 455 NAVAL POSTGRADUATE SCHOOL MONTEREY CA F/G 20/9
A COMPUTER ANALYSIS OF THE PLASMA-BOUNDARY LAYER BEHAVIOR OVER --ETC(U)
JUN 81 S T VAN BROCKLIN

UNCLASSIFIED NL




1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
MICROCOPY RESOLUTION TEST CHART

The Newton method involves approximating the nonlinear problem by a suitable linear problem involving derivatives of the nonlinear equations evaluated at $x^{(1)}$. The linearized problem is simply a set of linear algebraic equations, the solution to which furnishes an increment that when added to the solution $x^{(1)}$ will yield $x^{(2)}$. This method is a generalization to n-dimensions of Newton's method for evaluating the zeros of a function.

Each function $f_i(x)$ is approximated by the initial two terms of a Taylor series expansion about the solution vector $x^{(1)}$;

$$f_i(x) = f_i(x^{(1)}) + J(x^{(1)}) \cdot [x - x^{(1)}] \qquad (C4)$$

where "J" is the familiar Jacobian matrix,

$$J = \sum_{j=1}^{n} \frac{\partial f_i(x^{(1)})}{\partial x_j} \qquad (C5)$$

The right-hand-side of equation C4 is the linear vector function (tangent hyperplane) of x which best approximates the nonlinear function f evaluated at $x^{(1)}$. We set this right-hand-side equal to zero and solve for x, which is an improvement from $x^{(1)}$. We call this improved solution vector $x^{(2)}$, and from equation C4 we get;

$$x^{(2)} = x^{(1)} - J(x^{(1)})^{-1} \cdot f(x) \qquad (C6)$$

The Jacobian need not be known very accurately, and often is left unchanged for several iterations since its evaluation often entails penalizing computer time. Thus the method (slightly modified as described briefly in section III.B) used by Dolson [Ref. 12], is referred to in many texts as the Newton method or as the Newton-Raphson method [Refs. 25, 26].

The Jacobian matrix of coefficients for elliptic problems is typically a sparse banded matrix, to which there are many computer subroutines available for solution.

B. MODIFIED NEWTON-RAPHSON METHOD

The particular solution technique employed in this study merits a detailed discussion, since it is not common in the literature. Bailey and Touryan [Ref. 27] use a similar method and provide some description of its development. The method is similar to other Newton methods however, the Jacobian matrix is replaced by a matrix of coefficients developed from a sequence of linear differential equations which approximate the system of nonlinear equations. This method will be referred to as a modified Newton-Raphson method or MNR method for short, and will be presented in general using a specific example.

Take Poisson's equation given earlier and repeated here for simplicity;

$$\nabla^2\phi = c_1 (n_e - n_i) \qquad \text{(C7)}$$

The known solution variables are;

$$\phi^{(1)},\ n_e^{(1)},\ \text{and}\ n_i^{(1)}$$

In general the <u>first step</u> is linearization of the equation. In this case Eq. C7 <u>is linear</u>, but it will serve as an example. We assume that the suceeding solution variables at iteration-(2), namely $z^{(2)}$, can be approximated by the solution $z^{(1)}$ plus some small change $\Delta z^{(1)}$.

$$\Delta z^{(2)} = z^{(1)} + \quad z^{(1)} \qquad \text{(C8)}$$

where;

$$z^{(k)} = (\phi^{(k)}, n_e^{(k)}, n_i^{(k)})^T$$

It remains to determine $\Delta z^{(1)}$. Substitution of this relationship into the Poisson Equation is the <u>second step</u>;

$$\nabla^2\phi^{(2)} = \nabla^2\phi^{(1)} + \nabla^2\Delta\phi^{(1)} = c_1\left(n_e^{(1)} - n_i^{(1)}\right) + c_1\left(\Delta n_e^{(1)} - \Delta n_i^{(1)}\right)$$

rearranged;

$$\nabla^2\Delta\phi^{(1)} - c_1\left(\Delta n_e^{(1)} - \Delta n_i^{(1)}\right) = -\left[\nabla^2\phi^{(1)} - c_1\left(n_e^{(1)} - n_i^{(1)}\right)\right] \qquad \text{(C9)}$$

The right hand side of Eq. C9 is a function of the known solution vector $z^{(1)}$. We are applying this relationship at each computational node. A second order partial differential equation can be approximated by a second order difference equation or "3-point formula," so Eq. C9 will become a system of linear algebraic relationships involving near neighboring values of the solution vector $z^{(1)}$. The left side of Eq. C9 is composed of a system of coefficients expressible in terms of $z^{(1)}$ times the unknown (perturbation) vector $\Delta z^{(1)}$. So we now have;

$$A(z^{(1)}) \cdot \Delta z^{(1)} = -F(z^{(1)}) \qquad \text{(C10)}$$

Here the matrix of coefficients "A" is a sparse matrix with a relatively narrow band of non-zero elements about the main diagonal, provided that the "natural" order of the resulting linearized equations is preserved. There are standard computer routines available which take advantage of this banded structure, such as LEQT1B, an IMSL routine.

It follows that since $\Delta z^{(1)} = z^{(2)} - z^{(1)}$;

$$z^{(2)} = z^{(1)} - [A(z^{(1)})]^{-1} \cdot F(z^{(1)}) \qquad \text{(C11)}$$

For the case of a nonlinear term such as;

$$n_e \frac{\partial \phi}{\partial x}$$

there is no additional difficulty. When linearized this term becomes;

$$n_e \frac{\partial \phi}{\partial x} + \Delta n_e \frac{\partial \phi}{\partial x} + n_e \frac{\partial \Delta\phi}{\partial x} + \Delta n_e \frac{\partial \Delta\phi}{\partial x} \qquad \text{(C12)}$$

We neglect high order terms created by the inner products of the elements of $\Delta z^{(1)}$, such as the last term in relation C12. The first term would be a part of the function $F(z^{(1)})$ in the right side of Equation C10 , and the remaining two terms would be included as part of the expression:

$$A(z^{(1)}) \cdot \Delta z^{(1)}.$$

APPENDIX D

DESCRIPTION OF COMPUTER PROGRAMS

This appendix serves as a user oriented guide to the computer programs listed in App. E. The programs are listed in the order used to generate solutions to the problem formulated in this report. The output consists of printed results for each iteration and punched output to provide a permanent file copy, which is to be used as input to the three plotting programs also listed in App. E. These plotting programs use the Versatec plotter and associated software, and represent the only facility not commonly available elsewhere. The main program SHTH as presented, is configured to also produce output at a monitor terminal as used on CP/CMS (IBM 360) or MVS (IBM 370). The output in summary is via three devices;

DEVICE 4 - typically a CRT
DEVICE 6 - typically a printer
DEVICE 7 - typically a punch or disk output

These can be manipulated in convenient configurations to give a full or partial (sampled) display of the solution at each iteration, or merely to display convergence information in a simple format.

The SHTH program is the main program which together with its 9 subroutines and 1 IMSL subroutine solves the non-dimensionalized forms of Eqs. 1 through 4. The input to the program consists of control cards and data cards, namely;

1 control card
3 or 104 initial solution data cards
1 control card

The control card is described here in terms of the Fortran variable names thereon.

ITMIN - is the iteration counter (ITER) starting value. If ITMIN = 1, then the program will read in the next 3 cards as data which contain the short form starter solutions. If ITMIN is greater than 1, then the program will read in the next 104 cards as data, representing a detailed starting solution. This detailed starting solution is typically the output of a previous run of the program. If ITMIN = 00, then the program will not read in any data, being thus informed that the problem is over.

ITMAX - is the last iteration value. The program ceases iterations at this point and outputs the solutions and auxillary information.

ITNDT - enables the employment of the net production term (RTSIDE) when ITER is greater than ITNDT, otherwise the net production is set to zero.

ITEMP - for ITER less than ITEMP, the electron temperature is isothermal, otherwise it is a function of E/n. (see SHTH statements 899-990).

ITBC - for ITER greater than 5 and less than ITBC, the wall boundary conditions for the species is floated.

LADJ - 1 electron temperature is calculated
- 0 electron temperature is isothermal.

LPUN - 1 output is punched on device 7. - 0 output is not punched.

H - sets the mesh spacing equal to H*LAMS (sheath length).

FKTR - is the adjustment factor to the temperature equation (see Sec. IV) and is used when LADJ =1 and ITER is less than ITEMP. The adjustment factor is not used when LADJ is set to zero. When the temperature is coupled and LADJ = 1, then the program calculates its own adjustment factor.

NAME - is reserved for user labelling and identification which is used at output.

The solution set is;

PH voltage potential (volts).

NE electron density (10^{18} m^{-3})

NI ion density (10^{18} m^{-3})

TH nondimensional electron temperature

The short form starter solution is illustrated in App. E. It consists of PH, NE and NI values at the anode, wall and equilibrium boundaries. The long form starting solution consists of PH, NE, NI, and TH distributions, each one represented by 25 data cards and a label card. Each card has 7 values. (see format-803).

A typical iteration series would start out for about ten iterations from the short form data with ten volts at the anode and other appropriate boundary conditions. The initial solution converges very well for H = 0.5, the net production term coupled after 2 or 3 iterations, and the wall conditions not floated. This output is then used as input. The voltage is amplified to say 25 or 35 volts and a new control card is structured to couple the temperature or whatever else is desired. The H value may also be changed to reflect a smaller mesh spacing. It must be kept in mind that too many changes in one solution run can be catastrophic.

Each iteration of the solution by the SHTH program accomplishes the following;

(1) floats the NE, NI wall values

(2) sweeps through the A-matrix and C-vector setting up the equations that satisfy the boundary conditions and the internal mesh conditions

(3) couples the electron temperature equation

(4) couples the net production term

(5) calculates the anode electron density required to satisfy current matching btween the anode and the free-stream.

The system of equations has the form presented in Eq. C10 in App. C, and is solved by use of the IMSL routine LEQT1B. The solution vector is then updated from the return information contained in vector C.

There are two solution convergence monitors. The first is the sum of the squares of the differences (spectral norm) of the $-F(z^{(i)})$ values from Eq. C10. The other is the spectral norm of the $\Delta z^{(i)}$ upon return from LEQT1B. Both of these are available at the video terminal (device 4) and printer (device 6).

After solution update, the elctron temperature is calculated and adjusted per the control card options. It should

be pointed out that on the last iteration, the temperature distribution is automatically calculated for output, but is not used in the scheme, unless otherwise coupled per the control card.

The matching of the anode current to the free-stream current is accomplished by solving for the anode electron density using subroutine **ANODE**. The call to this subprogram will require a change to the **SHTH** program as it is presently overridden with a comment card, (prior to statement 9000).

The final outputs are straight-forward. Subroutines **JWALL** and **JOULE** are called in this phase of the program, in order to compute the current into the wall and the Joule heat distribution. The electric field, net production and space charge density distributions are also calculated and provided as punched and printed output.

The punched output is used as input for the **STREAM, PICT** and **PLOT** programs. These programs use the Versatec plotter to provide a presentation of the current stream lines, an oblique perspectives of the solutions and plots of the solutions as they vary normal to the wall extending from the anode site. The details of these programs are cumbersome, and it is wise to leave them to the user for inspection.

The first two distributions in the punched output are used in the STREAM program, requiring that the nonpertinent data be removed. The next 8 distributions are used for the PLOT and PICT programs. The PICT program has an option card filled with ones or zeros in the first eight positions. This option card enables the user to elect which distributions to plot. The plots provided in this report as Figs. 12 through 14, were produced using the PICT and PLOT programs. The programs are simple and speak for themselves.

APPENDIX E

COMPUTER PROGRAM LISTINGS

This appendix contains a listing of the programs used in the solution of the equations presented in this work (namely: Eqs. 9,10 and 11), and the auxiliary programs that provided the plots for the report.

```
C                                                                       SHT00010
C                                                                       SHT00020
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCSHT00030
C                                                                       SHT00040
C                                                                       SHT00050
C                          SHTH                                         SHT00060
C                                                                       SHT00070
C                       (MAIN PROGRAM)                                  SHT00080
C                                                                       SHT00090
C                        SIZE  25 X 7                                   SHT00100
C                                                                       SHT00110
C                                                                       SHT00120
C                                                                       SHT00130
C                                                                       SHT00140
C                                                                       SHT00150
C                             S.T. VAN BROCKLIN, LCDR,USN               SHT00160
C                                                                       SHT00170
C                             DEPT OF AERONAUTICS                       SHT00180
C                             NAVAL POST GRADUATE SCHOOL                SHT00190
C                             MONTEREY, CALIFORNIA                      SHT00200
C                                                                       SHT00210
C                             APRIL    1981                             SHT00220
C                                                                       SHT00230
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCSHT00240
C                                                                       SHT00250
C                          ** NOTES **                                  SHT00260
C                                                                       SHT00270
C                                                                       SHT00280
C       1  RTSIDE COUPLED IF ITER.GT.ITNDT                              SHT00290
C                                                                       SHT00300
C       2  THETA COUPLED IF ITER.GT.ITGRD                               SHT00310
C          THETA=1 OTHERWISE, EXCEPT COUPLED AT LAST ITERATION          SHT00320
C          FOR DISPLAY OF PROFILE ONLY                                  SHT00330
C                                                                       SHT00340
C       3  WALL PERPENDICULAR CURRENT  (JX=0) B.C.                      SHT00350
C          SATISFIED WITH ELECTRON DENSITY GRADIENTS ONLY               SHT00360
C          E.G. NEGLECT ION DENSITIES.                                  SHT00370
C                                                                       SHT00380
C       4  NE AND NI DENSITIES ALONG THE WALL ARE "FLOATED"             SHT00390
C          TO THE LOWEST OF NEAREST VALUES WITHIN 6 COMPUTATIONAL       SHT00400
C          NODES AWAY FROM WALL.                                        SHT00410
C          THIS IS ACCOMPLISHED FROM ITER = NBC TO ITBC.                SHT00420
C                                                                       SHT00430
C       5  X-COORD IS DISTANCE FROM THE WALL                            SHT00440
C          Y-COORD IS DISTANCE ALONG THE WALL                           SHT00450
C            E.G. NE(I,J)=ELECTRON DENSITY AT (X,Y)                     SHT00460
C                                                                       SHT00470
C                                                                       SHT00480
```

```
C                                                                       SHT00490
C                                                                       SHT00500
C                                                                       SHT00510
C                     ** DESCRIPTION **                                 SHT00520
C                                                                       SHT00530
C                                                                       SHT00540
C     THIS PROGRAM SOLVES THE SHEATH AND AMBIPOLAR REGION OF THE        SHT00550
C     ELECTRODE BOUNDARY LAYER IN A HIGH PRESSURE, HIGH ENERGY          SHT00560
C     DENSITY ELECTRICAL DISCHARGE IN NITROGEN UNDER GLOW DISCHARGE     SHT00570
C     CONDITIONS.                                                       SHT00580
C                                                                       SHT00590
C     APPLICATION IS FOR ELECTRON DENSITIES OF 1.E+18 PER CU-METER,     SHT00600
C     FOR E/N=1.E-16 VOLTS-CM-SQ, AT A GAS DENSITY CORRESPONDING        SHT00610
C     TO ONE AMAGAT.                                                    SHT00620
C                                                                       SHT00630
C     THE WORKING EQUATIONS ARE NON-LINEAR, COUPLED AND SECOND ORDER.   SHT00640
C          (1) POISSON EQUATION                                         SHT00650
C          (2) ELECTRON CONSERVATION                                    SHT00660
C          (3) ION CONSERVATION                                         SHT00670
C     THE ELECTRON ENERGY IS COMPUTED FROM THE LOCAL ELECTRIC FIELD.    SHT00680
C                                                                       SHT00690
C     THE PROGRAM CONSISTS OF THE FOLLOWING ....                        SHT00700
C          MAIN ROUTINE: INITIALIZES, CONSTRUCTS MATRIX OF COEF,        SHT00710
C                        UPDATES SOLUTIONS, AND CONTROLS OVERALL        SHT00720
C                        PROGRAM OPTIONS USING INPUTED INDICES.         SHT00730
C               SUBR LEQT1B: AN IMSL ROUTINE, THAT FOR A GIVEN N*N      SHT00740
C                        BANDED,REAL MATRIX, FACTORS THE MATRIX A       SHT00750
C                        INTO THE L-U DECOMPOSITION OF A ROW WISE       SHT00760
C                        PERMUTATION OF A AND/OR SOLVES THE SYSTEM      SHT00770
C                        OF EQUATIONS  AX=B.                            SHT00780
C               SUBR DELOP:  COMPUTES DIFFERENCES OF SOLU AT IC,JC      SHT00790
C               SUBR GRDNT:  COMPUTES THE GRADIENT OF THE INPUTED FIELD SHT00800
C                        VALUE IN BOTH COORDS AND THE SCALER            SHT00810
C               SUBR TEMP:   COMPUTES THE ELECTRON TEMP DIST            SHT00820
C               SUBR RTSIDE: CALCULATES THE NE, NI PRODUCTION (ION/RECOMB) SHT00830
C               SUBR PATCH:  USED TO BOUND SOLUTIONS NOT PHYS COMPAT.   SHT00840
C               SUBR ANODE:  CALCULATES JEQUI AND JNODE                 SHT00850
C                        CALCULATES NE(ANODE) REQUIRED FOR JEQUI=JNODE  SHT00860
C               SUBR JWALL:  CALCULATES THE CURRENT INTO THE WALL       SHT00870
C               SUBR JOULE:  COMPUTES THE JOULE HEATING  J-E            SHT00880
C               SUBR OUTPUT: SENDS RESULTS TO ANY DESIGNATED OUTPUT DEVICE SHT00890
C                                                                       SHT00900
C                                                                       SHT00910
C     THE MAIN ROUTINE IS BROKEN INTO SEVERAL SECTIONS BY PERTINENT     SHT00920
C     COMMENT STATEMENTS.  THE PROGRAM INITIALIZES THE WORKING CONSTANTS SHT00930
C     AND VARIABLES FROM THE INPUTED DATA.  INPUT CONSISTS OF A CONTROL SHT00940
C     CARD, CONSISTING OF ITERATION START, STOP VALUES, CONTROL         SHT00950
C     INDICES, AND SHEATH SIZE NUMBER.  SUBSEQUENT CARDS                SHT00960
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C     ARE EITHER A COMPLETE STARTING SOLUTION OF  POTENTIAL,                SHT00970
C     NE, AND NI DENSITIES; OR AN ABREVIATED BOUNDARY CONDITION             SHT00980
C     DATA SET.  THE VARIABLE ITMIN (FIRST CARD) DICTATES THE               SHT00990
C     FORMAT OF THE SUBSEQUENT INPUT DATA.                                  SHT01000
C                                                                           SHT01010
C     THE A MATRIX REPRESENTS A SYSTEM OF COEFFICIENTS WHICH ARE            SHT01020
C     THEMSELVES FUNCTIONS OF THE KNOWN SOLUTION VARIABLES:                 SHT01030
C                        PH, NE, NI, TH                                     SHT01040
C     AS DESCRIBED IN THE REPORT.  THE A MATRIX IS STORED IN A SPACE        SHT01050
C     ECONOMIZED BANDED MODE.                                               SHT01060
C     THE MATRIX COEFFICIENTS ARE CALCULATED IN THREE SWEEPS,               SHT01070
C     (M=1,2, OR 3) CORRESPONDING TO THE POISSON EQUATION,                  SHT01080
C     ELECTRON AND ION SPECIES CONSERVATION EQUATIONS, AT                   SHT01090
C     EACH OF THE  COMPUTATIONAL NODES.                                     SHT01100
C     THESE ARE COMPUTED FOR THE WALL (JX=0) B.C. (DO LOOP 200),            SHT01110
C     THE BOUNDED AREA (DO LOOP 500), AND THE REMAINING                     SHT01120
C     B.C.'S (SERIES 500 STMNTS).                                           SHT01130
C     THE INTERIOR NODES ARE CALCULATED AT STMNTS 410,420,430               SHT01140
C                                                                           SHT01150
C     THE BANDED MATRIX AND ASSOCIATED SOLUTION VECTOR ARE SOLVED USING     SHT01160
C     THE IMSL SUBROUTINE LEQT1B.                                           SHT01170
C                                                                           SHT01180
C     UPON CALLING LEQT1B THE C-VECTOR CONTAINS THE                         SHT01190
C     FUNCTIONAL RELATIONS -F(PH,NE,NI,TH) FOR THE POISSON AND              SHT01200
C     ELECTRON AND ION CONSERVATION EQUATIONS.                              SHT01210
C                                                                           SHT01220
C     THE C- VECTOR UPON RETURNING TO THE MAIN PROGRAM FROM                 SHT01230
C     LEQT1B, REPRESENTS A CORRECTION VECTOR TO THE PREVIOUS                SHT01240
C     SOLUTION.  THE PH,NE, AND NI MATRICES ARE UPDATED                     SHT01250
C     (SERIES 700 STMNTS), AND THE TEMPERATURE (TH MATRIX)                  SHT01260
C     IS COMPUTED FROM THE POTENTIAL GRADIENT (SERIES 900 STMNTS)           SHT01270
C                                                                           SHT01280
C     ALL OF THE ANCILIARY SUBROUTINES ARE STRAIGHTFORWARD,                 SHT01290
C     MERITING NO ADDITIONAL EXPLANATION.                                   SHT01300
C                                                                           SHT01310
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCSHT01320
C                                                                           SHT01330
C                        ** VARIABLES **                                    SHT01340
C                                                                           SHT01350
C                                                                           SHT01360
C                                                                           SHT01370
C                                                                           SHT01380
C        ALFA#    RECOMBINATION COEFFICIENTS                               SHT01390
C        ASTAR    ELECTRAN DENSITY AT INFINITY  (1/METER**3)               SHT01400
C        ANEUT    GAS DENSITY AT INFINITY  (1/METER**3)                    SHT01410
C        DEO      DIFFUSION COEF FOR ELECTRONS IN NITROGEN                 SHT01420
C        DIO      DIFFUSION COEF FOR NITROGEN  (N+ IN N2)                  SHT01430
C        ITMIN    STARTING ITERATION VALUE                                 SHT01440
```

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C          ITMAX    ENDING ITERATION VALUE                                          SHT01450
C                      NUMBER OF ITERATIONS=ITMAX-ITMIN +1                         SHT01460
C          ITNDT    NEDOT IS COUPLED AFTER ITNDT ITERATIONS                        SHT01470
C          ITGRD    ELECTRON ENERGY IS COUPLED AFTER ITGRD ITERATIONS               SHT01480
C          ITBC     ENABLES FLOATING WALL B.C. FOR ITER.LE.ITBC                     SHT01490
C          NPUN     PUNCHED OUTPUT IF NPUNCH=01                                    SHT01500
C          H        COMP. MOLECULE INTERVAL (= NUM LAMS PER SEPARATION)            SHT01510
C          FKTR     SETS A CONSTANT VALUE FOR 'ADJ' WHEN 'LADJ=0'                  SHT01520
C          PHIZ     NODE POTENTIAL (VOLTS)                                         SHT01530
C          PHW      STARTING POTENTIAL ALONG WALL (DUMMY VARB, NOT USED)           SHT01540
C          PHQ      POTENTIAL B.C. AT FREESTREAM                                   SHT01550
C          NED      ELECTRON DENSITY AT NODE                                       SHT01560
C          NID      ION DENSITY AT NODE                                            SHT01570
C          NEW      ELECTRON DENSITY ALONG WALL                                    SHT01580
C          NIW      ION DENSITY ALONG WALL                                         SHT01590
C          NEQ      ELECTRON DENSITY AT FREESTREAM                                 SHT01600
C          NIQ      ION DENSITY AT FREESTREAM                                      SHT01610
C          S####    THESE VARIABLES ARE SAVED FOR OUTPUT                           SHT01620
C          RECOM    COMBINATION COEF FOR SIMPLIFICATION                            SHT01630
C          BETA     TEMP(ELECT)/TEMP(GAS)                                          SHT01640
C          LAMS     SHEATH LENGTH  (METERS)                                        SHT01650
C          EQP      NONDIMENSIONAL CONSTANT FOR POISSON EQN.                       SHT01660
C          EQE      NONDIMENSIONAL CONSTANT FOR ELECTRON DENSITY EQN.              SHT01670
C          EQI      NONDIMENSIONAL CONSTANT FOR ION DENSITY EQN.                   SHT01680
C          EQT      DIMENSIONAL CONSTANT TO NORMALIZE E/N FOR TEMP UPDATE          SHT01690
C          ESP      =EQP*H  ETC. FOR EQE,EQI                                       SHT01700
C          PH       NONDIMENSIONED POTENTIAL FIELD                                 SHT01710
C          NE       NONDIMENSIONED ELECTRAN DENSITY                                SHT01720
C          NI       NONDIMENSIONED ION DENSITY                                     SHT01730
C          TH       NONDIMENSIONED  T(EL)/T(EL@INF)                                SHT01740
C          DPHDX    POTENTIAL GRADIENT ALONG X                                     SHT01750
C                     SIMILAR FOR NE,NI,TH                                         SHT01760
C          GRAD     SCALER GRADIENT = SQRT(GRDX**2+GRDY**2)                        SHT01770
C          HEAT     JOULE HEAT ARRAY FROM SUBR: JOULE                              SHT01780
C          NEDOT    NET PRODUCTION TERM FROM SUBR: RTSIDE                          SHT01790
C          JNODE    CURRENT INTO THE ACTIVE SITE                                   SHT01800
C          JEQUI    FREE STREAM CURRENT TOTAL                                      SHT01810
C                                                                                   SHT01820
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCSHT01830
C                                                                                   SHT01840
C                                                                                   SHT01850
C                                                                                   SHT01860
C                                                                                   SHT01870
C                                                                                   SHT01880
C                                                                                   SHT01890
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCSHT01900
C                                                                                   SHT01910
C                    MAIN PROGRAM                                                   SHT01920
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C                    810408                                                    SHT01930
CCC                                                                            SHT01940
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCSHT01950
CCC                                                                            SHT01960
CCC                                                                            SHT01970
C                                                                              SHT01980
      IMPLICIT REAL*8(A-H,O-$)                                                 SHT01990
      EXTERNAL LEQT1B,DELOP,RTSIDE,JWALL,PATCH,OUTPUT,GRDNT,JOULE,ANODE        SHT02000
      REAL*8 A(525,46),C(525),XL(13000)                                        SHT02010
      REAL*8 PH(25,7),NE(25,7),NI(25,7),TH(25,7),NEDOT(25,7)                   SHT02020
      REAL*8 GRDX(25,7),GRDY(25,7),GRAD(25,7),HEAT(25,7)                       SHT02030
      REAL*8 NED,NEW,NEQ,NID,NIW,NIQ,LAMS,JNODE,JEQUI                          SHT02040
      REAL*4 NAME(10),DUMMY(20)                                                SHT02050
      COMMON /CURT/ DEO,DIO,LAMS,BETA                                          SHT02060
      COMMON /EQUATS/ CONE,CONJ,EQP,EQE,EQI,EQT                                SHT02070
      COMMON /IONIZ/ RECOM,ALFA1,ALFA2,ALFA3,TEINF                             SHT02080
      COMMON /IPARM/ NC,NS,ITER                                                SHT02090
      COMMON /DPARM/ M,IC,JC,ICM,ICP,JCM,JCP                                   SHT02100
      COMMON /DIVER/ DPHDX,DPHDY,DNEDX,DNEDY,DNIDX,DNIDY,DTHDX,DTHDY           SHT02110
C-------------------------------------------------------------------------     SHT02120
CCC                                                                            SHT02130
CCC                  INITIALIZE PARAMETERS                                     SHT02140
CCC                                                                            SHT02150
C-------------------------------------------------------------------------     SHT02160
      NC=25                                                                    SHT02170
      NS=7                                                                     SHT02180
      NSY3=NS*3                                                                SHT02190
      NXN=NC*NSY3                                                              SHT02200
      NXNM=NXN-NSY3                                                            SHT02210
      III=NXN                                                                  SHT02220
      JJJ=2*NSY3+4                                                             SHT02230
      J1=3                                                                     SHT02240
      J2=NSY3                                                                  SHT02250
      JD=J2+3                                                                  SHT02260
      J4=JD+3                                                                  SHT02270
      J5=2*NSY3+3                                                              SHT02280
C-------------------------------------------------------------------------     SHT02290
CCC                                                                            SHT02300
CCC                  SET UP CONSTANTS                                          SHT02310
CCC                                                                            SHT02320
C-------------------------------------------------------------------------     SHT02330
      EPSO=8.854D-12                                                           SHT02340
      COUL=1.602192D-19                                                        SHT02350
      BOLTZ=1.380622D-23                                                       SHT02360
      ALFA1=1.5D-12                                                            SHT02370
      ALFA2=1.09D-02                                                           SHT02380
      ALFA3=0.74D-38                                                           SHT02390
      DENSO=2.7D+25                                                            SHT02400
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      DENSE=1.D+18                                                          SHT02410
      DEO=6.65D-02                                                          SHT02420
      DID=8.00D-06                                                          SHT02430
      RECOM=ALFA1+ALFA3*DENSO                                               SHT02440
      BETA=+38.5D0                                                          SHT02450
      TGAS=273.D0                                                           SHT02460
      TEINF=TGAS*BETA                                                       SHT02470
      UPH=BOLTZ*BETA*TGAS/COUL                                              SHT02480
    1 CONTINUE                                                              SHT02490
      DT=0.D0                                                               SHT02500
      ITER=0                                                                SHT02510
      ADJ=1.D0                                                              SHT02520
C-----------------------------------------------------------------------    SHT02530
CC                                                                          SHT02540
CCC                                                                         SHT02550
CCC                                                                         SHT02560
C-----------------------------------------------------------------------    SHT02570
      DO 50 INC=1,NC                                                        SHT02580
      DO 50 JNC=1,NS                                                        SHT02590
      PH(INC,JNC)=0.D0                                                      SHT02600
      NE(INC,JNC)=1.D0                                                      SHT02610
      NI(INC,JNC)=1.D0                                                      SHT02620
      GRDX(INC,JNC)=0.D0                                                    SHT02630
      GRDY(INC,JNC)=0.D0                                                    SHT02640
      GRAD(INC,JNC)=0.D0                                                    SHT02650
      NEDOT(INC,JNC)=0.D0                                                   SHT02660
      HEAT(INC,JNC)=0.D0                                                    SHT02670
   50 TH(INC,JNC)=1.D0                                                      SHT02680
      DO 51 JKL=1,NS                                                        SHT02690
      NE(1,JKL)=0.D0                                                        SHT02700
      NI(1,JKL)=0.D0                                                        SHT02710
      NE(NC,JKL)=0.D0                                                       SHT02720
      NI(NC,JKL)=0.D0                                                       SHT02730
   51 CONTINUE                                                              SHT02740
      READ(5,800) ITMIN,ITMAX,ITNDT,ITEMP,ITBC,LADJ,LPUN,H,FKTR,NAME        SHT02750
      IF(ITMIN.LE.0) GO TO 9999                                             SHT02760
C                                                                           SHT02770
CC          SET THE TEMP ADJ FKTR                                           SHT02780
C                                                                           SHT02790
      IF(LADJ.EQ.0) ADJ=FKTR                                                SHT02800
      WRITE(4,847) ITMIN,ITMAX,ITNDT,ITEMP,ITBC,LADJ,LPUN,H,FKTR,NAME       SHT02810
      WRITE(6,847) ITMIN,ITMAX,ITNDT,ITEMP,ITBC,LADJ,LPUN,H,FKTR,NAME       SHT02820
      IF(ITMAX.LE.ITNDT) WRITE(6,850)                                       SHT02830
      IF(ITMAX.GT.ITNDT) WRITE(6,851) ITNDT                                 SHT02840
      IF(ITMAX.GT.ITEMP) WRITE(6,853) ITEMP                                 SHT02850
      IF(LADJ.GT.0) WRITE(6,881)                                            SHT02860
      IF(LADJ.EQ.0) WRITE(6,882)                                            SHT02870
      IF(ITMAX.LE.ITEMP) WRITE(6,852)                                       SHT02880
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      IF(ITMIN.GE.ITBC.OR.ITBC.LE.NBC) WRITE(6,854)                          SHT02890
      IF(ITBC.GT.ITMIN.AND.ITBC.GT.NBC) WRITE(6,855) NBC,ITBC                SHT02900
      IF(ITMIN.GT.1) GO TO 10                                                SHT02910
      READ(5,801) PHIZ,PHW,PHQ                                               SHT02920
      READ(5,801) NED,NEW,NEQ                                                SHT02930
      READ(5,801) NID,NIW,NIQ                                                SHT02940
      WRITE(6,811) PHIZ,PHW,PHQ                                              SHT02950
      WRITE(6,812) NED,NEW,NEQ                                               SHT02960
      WRITE(6,813) NID,NIW,NIQ                                               SHT02970
      WRITE(4,811) PHIZ,PHW,PHQ                                              SHT02980
      WRITE(4,812) NED,NEW,NEQ                                               SHT02990
      WRITE(4,813) NID,NIW,NIQ                                               SHT03000
      SNED=NED                                                               SHT03010
      SNEW=NEW                                                               SHT03020
      SNEQ=NEQ                                                               SHT03030
      SNID=NID                                                               SHT03040
      SNIW=NIW                                                               SHT03050
      SNIQ=NIQ                                                               SHT03060
      SPHW=PHW                                                               SHT03070
      SPHQ=PHQ                                                               SHT03080
      GO TO 20                                                               SHT03090
   10 CONTINUE                                                               SHT03100
      READ (5,803) DUMMY                                                     SHT03110
      DO 11 I=1,NC                                                           SHT03120
   11 READ(5,804)  (PH(I,J),J=1,NS)                                          SHT03130
      READ(5,803) DUMMY                                                      SHT03140
      DO 12 I=1,NC                                                           SHT03150
   12 READ(5,804)  (NE(I,J),J=1,NS)                                          SHT03160
      READ(5,803) DUMMY                                                      SHT03170
      DO 13 I=1,NC                                                           SHT03180
   13 READ(5,804)  (NI(I,J),J=1,NS)                                          SHT03190
      READ(5,803) DUMMY                                                      SHT03200
      DO 14 I=1,NC                                                           SHT03210
   14 READ(5,804)  (TH(I,J),J=1,NS)                                          SHT03220
      SNED=NE(1,1)                                                           SHT03230
      SNID=NI(1,1)                                                           SHT03240
      PHIZ=PH(1,1)                                                           SHT03250
      DO 18 J=1,NS                                                           SHT03260
      DO 18 I=1,NC                                                           SHT03270
   18 PH(I,J)=PH(I,J)/UPH                                                    SHT03280
      CALL  PATCH(TH,1.D0,99.D0)                                             SHT03290
      CALL OUTPUT(6,PH,UPH,'VOLTAGE POTENTIAL        ')                     SHT03300
      CALL OUTPUT(6,NE,1.D0,'ELECTRON DENSITY        ')                     SHT03310
      CALL OUTPUT(6,NI,1.D0,'ION DENSITY             ')                     SHT03320
      CALL OUTPUT(6,TH,1.D0,'TEMPERATURE DISTRIBUTION')                      SHT03330
   20 CONTINUE                                                               SHT03340
C-----------------------------------------------------------------------    SHT03350
C                                                                           SHT03360
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C                  SET UP NONDIMENSIONAL VALUES                                SHT03370
C                                                                              SHT03380
C-------------------------------------------------------------------------------SHT03390
      HH=H*H                                                                   SHT03400
      PHIB=PHIZ/UPH                                                            SHT03410
      SPHIB=PHIB                                                               SHT03420
      LAMS=(EPSO*PHIZ/(COUL*DENSE))**.5DO                                      SHT03430
      CONE=BOLTZ*TEINF/(COUL*LAMS)                                             SHT03440
      CONJ=COUL*DENSE/LAMS                                                     SHT03450
      EQP=COUL*PHIZ/(BOLTZ*BETA*TGAS)*HH                                       SHT03460
      EQE=DENSE*LAMS*LAMS*HH/DEO                                               SHT03470
      EQI=EQE*DEO/DIO                                                          SHT03480
      EQT=UPH/(LAMS*DENSO)*1.D+20/H                                            SHT03490
C-------------------------------------------------------------------------------SHT03500
C                                                                              SHT03510
C                  BEGIN ITERATIONS                                            SHT03520
C                                                                              SHT03530
C-------------------------------------------------------------------------------SHT03540
      DO 9000 ITER=ITMIN,ITMAX                                                 SHT03550
      WRITE(4,808)                                                             SHT03560
      WRITE(6,808)                                                             SHT03570
      IF(ITER.GT.1) NIW=0.DO                                                   SHT03580
      IF(ITER.GT.1) NEW=0.DO                                                   SHT03590
      IF(ITER.GT.1) NEQ=0.DO                                                   SHT03600
      IF(ITER.GT.1) NIQ=0.DO                                                   SHT03610
      IF(ITER.GT.1) NED=0.DO                                                   SHT03620
      IF(ITER.GT.1) NID=0.DO                                                   SHT03630
      IF(ITER.GT.1) PHIB=0.DO                                                  SHT03640
      IF(ITER.GT.1) PHW=0.DO                                                   SHT03650
      IF(ITER.GT.1) PHQ=0.DO                                                   SHT03660
      IF(ITER.GT.ITNDT) DT=1.DO                                                SHT03670
      IF(ITER.GT.ITNDT) CALL RTSIDE(NE,NI,TH,NEDOT)                            SHT03680
C-------------------------------------------------------------------------------SHT03690
C                                                                              SHT03700
C                  FLOAT WALL NE,NI B.C.                                       SHT03710
C                                                                              SHT03720
C                  NE,NI TAKE ON NEAREST NEIGHBOR MINIMUM VALUES               SHT03730
C                  OUT TO 6 COMPUTATIONAL NODES FROM THE WALL.                 SHT03740
C                                                                              SHT03750
C                  FLOAT WALL B.C. FOR ITER.GT.NBC AND ITER.LT.ITBC            SHT03760
C                                                                              SHT03770
C-------------------------------------------------------------------------------SHT03780
      NBC=5                                                                    SHT03790
      IF(ITER.LT.NBC) GO TO 90                                                 SHT03800
      IF(ITER.GT.ITBC) GO TO 90                                                SHT03810
      DO 88 JX=2,NS                                                            SHT03820
      XNEW=1.DO                                                                SHT03830
      XNIW=1.DO                                                                SHT03840
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      DO 85 IX=1,6                                                      SHT03850
CR    DO 85 INX=1,5                                                     SHT03860
CR    IX=7-INX                                                          SHT03870
CR    IF(NE(IX-1,JX).GT.NE(IX,JX)) NE(IX-1,JX)=NE(IX,JX)                SHT03880
CR    IF(NI(IX-1,JX).GT.NI(IX,JX)) NI(IX-1,JX)=NI(IX,JX)                SHT03890
      IF(NE(IX,JX).LE.XNEW) NEM=IX                                      SHT03900
      IF(NE(IX,JX).LE.XNEW) XNEW=NE(IX,JX)                              SHT03910
      IF(NI(IX,JX).LE.XNIW) NIM=IX                                      SHT03920
      IF(NI(IX,JX).LE.XNIW) XNIW=NI(IX,JX)                              SHT03930
   85 CONTINUE                                                          SHT03940
      DO 86 IX=1,NEM                                                    SHT03950
      NE(IX,JX)=XNEW                                                    SHT03960
   86 CONTINUE                                                          SHT03970
      DO 87 IX=1,NIM                                                    SHT03980
   87 NI(IX,JX)=XNIW                                                    SHT03990
   88 CONTINUE                                                          SHT04000
   90 CONTINUE                                                          SHT04010
      CALL PATCH(NE,0.D0,1.D0)                                          SHT04020
      CALL PATCH(NI,0.D0,1.D0)                                          SHT04030
C-----------------------------------------------------------------------SHT04040
C                                                                       SHT04050
C                   SET UP COEFFICIENTS FOR MATRIX...A                  SHT04060
C                                       AND VECTOR...C                  SHT04070
C                                                                       SHT04080
C                                                                       SHT04090
C                   M.EQ.1   POISSON EQUATION                           SHT04100
C                   M.EQ.2   ELECTRON CONCENTRATION                     SHT04110
C                   M.EQ.3   ION CONCENTRATION                          SHT04120
C                                                                       SHT04130
C-----------------------------------------------------------------------SHT04140
      I1=3                                                              SHT04150
      I2=NSY3-3                                                         SHT04160
      DO 100 INC=1,III                                                  SHT04170
      C(INC)=0.D0                                                       SHT04180
      DO 100 JNC=1,JJJ                                                  SHT04190
  100 A(INC,JNC)=0.D0                                                   SHT04200
      DO 5000 M=1,3                                                     SHT04210
      I1=I1+1                                                           SHT04220
      I2=I2+1                                                           SHT04230
C-----------------------------------------------------------------------SHT04240
C                                                                       SHT04250
C                     SET UP FIRST 21 ROWS (WALL B.C.)                  SHT04260
C                                                                       SHT04270
C-----------------------------------------------------------------------SHT04280
      DO 200 IR=I1,I2,3                                                 SHT04290
      IF(M.NE.1) GO TO 190                                              SHT04300
C-----------------------------------------------------------------------SHT04310
C                                                                       SHT04320
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C                    ZERO CURRENT INTO WALL  FOR M.EQ.1                          SHT04330
C                                                                                SHT04340
C--------------------------------------------------------------------------------SHT04350
      DFT=DEO/DIO                                                                SHT04360
      XIR=DFLOAT(IR+2)                                                           SHT04370
      JB=XIR/3.DO                                                                SHT04380
C3    DPHDX=(-3.DO*PH(1,JB)+4.DO*PH(2,JB)-Ph(3,JB))/2.DO                        SHT04390
C3    DNEDX=(-3.DO*NE(1,JB)+4.DO*NE(2,JB)-NE(3,JB))/2.DO                        SHT04400
C3    DNIDX=(-3.DO*NI(1,JB)+4.DO*NI(2,JB)-NI(3,JB))/2.DO                        SHT04410
      DPHDX=PH(2,JB)-PH(1,JB)                                                    SHT04420
      DNEDX=NE(2,JB)-NE(1,JB)                                                    SHT04430
      DNIDX=NI(2,JB)-NI(1,JB)                                                    SHT04440
      IF(ITER.EQ.1) DPHDX=0.DO                                                   SHT04450
      IF(ITER.EQ.1) DNEDX=0.DO                                                   SHT04460
      IF(ITER.EQ.1) DNIDX=0.DO                                                   SHT04470
CK    A(IR,JD)=+NE(1,JB)/TH(1,JB)*DFT+BETA*NI(1,JB)                              SHT04480
CK    IF(ITER.EQ.1) A(IR,JD)=+NEW/TH(1,JB)*DFT+BETA*NIW                          SHT04490
CK    A(IR,J5)=-A(IR,JD)                                                         SHT04500
CK    A(IR,JD+1)=-DFT*DPHDX/TH(1,JB)-DFT                                         SHT04510
CK    A(IR,J5+1)=+DFT                                                            SHT04520
CK    A(IR,JD+2)=-BETA*DPHDX+1.DO                                                SHT04530
CK    A(IR,J5+2)=-1.DO                                                           SHT04540
CK    CIRCLE=DFT*(NE(1,JB)*DPHDX/TH(1,JB)-DNEDX)                                 SHT04550
CK    C(IR)=CIRCLE+BETA*NI(1,JB)*DPHDX+DNIDX                                     SHT04560
      WRITE(6,753) DPHDX,DNEDX,DNIDX                                             SHT04570
  753 FORMAT(3X,3(D10.4,3X))                                                     SHT04580
      A(IR,JD)=+NE(1,JB)                                                         SHT04590
      IF(ITER.EQ.1) A(IR,JD)=NEW                                                 SHT04600
      A(IR,J5)=-A(IR,JD)                                                         SHT04610
      A(IR,JD+1)=-DPHDX-TH(1,JB)                                                 SHT04620
      A(IR,J5+1)=+TH(1,JB)                                                       SHT04630
      C(IR)=NE(1,JB)*DPHDX-TH(1,JB)*DNEDX                                        SHT04640
  190 IF(M.NE.1) A(IR,JD)=1.DO                                                   SHT04650
      IF(M.EQ.2) C(IR)=NEW                                                       SHT04660
      IF(M.EQ.3) C(IR)=NIW                                                       SHT04670
  200 CONTINUE                                                                   SHT04680
C--------------------------------------------------------------------------------SHT04690
C                                                                                SHT04700
C                  SET UP ROWS 22 THRU NXN-22  (FIELD AND SYMMETRY)              SHT04710
C                                                                                SHT04720
C--------------------------------------------------------------------------------SHT04730
      ISTART=NSY3+M                                                              SHT04740
      IEND=NXNM-3+M                                                              SHT04750
      IC=1                                                                       SHT04760
      JC=1                                                                       SHT04770
      DO 500 IR=ISTART,IEND,3                                                    SHT04780
      CALL DELOP(PH,NE,NI,TH)                                                    SHT04790
      THETA=TH(IC,JC)                                                            SHT04800
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      ALP=1.09D-02*(THETA*BETA*TGAS)**(-4.5DC)                                  SHT04810
      GO TO (410,420,430),M                                                     SHT04820
C-----------------------------------------------------------------------------SHT04830
C                                                                               SHT04840
C                    ARRIVE HERE FOR M.EQ.1 (POISSON EQUATION)                  SHT04850
C                                                                               SHT04860
C-----------------------------------------------------------------------------SHT04870
  410 CONTINUE                                                                  SHT04880
      A(IR,J1)=-1.D0                                                            SHT04890
      A(IR,J2)=-1.D0                                                            SHT04900
      A(IR,JD)=+4.D0                                                            SHT04910
      A(IR,J4)=-1.D0                                                            SHT04920
      A(IR,J5)=-1.D0                                                            SHT04930
      A(IR,JD+1)=+EQP                                                           SHT04940
      A(IR,JD+2)=-EQP                                                           SHT04950
      C(IR)=+(PH(ICP,JC)+PH(ICM,JC)+PH(IC,JCP)+PH(IC,JCM)-4.D0*PH(IC,JC)SHT04960
     1       - EQP*(NE(IC,JC)-NI(IC,JC)))                                       SHT04970
      GO TO 450                                                                 SHT04980
C-----------------------------------------------------------------------------SHT04990
C                                                                               SHT05000
C                    ARRIVE HERE FOR M.EQ.2 (ELECTRON CONCEN)                   SHT05010
C                                                                               SHT05020
C-----------------------------------------------------------------------------SHT05030
  420 CONTINUE                                                                  SHT05040
      A(IR,J1-1)=+.5D0*DNEDX-.5D0*NE(IC,JC)*DTHDX/THETA                         SHT05050
      A(IR,J2-1)=+.5D0*DNEDY-.5D0*NE(IC,JC)*DTHDY/THETA                         SHT05060
      A(IR,J4-1)=-A(IR,J2-1)                                                    SHT05070
      A(IR,J5-1)=-A(IR,J1-1)                                                    SHT05080
      A(IR,J1)= +.5D0*DPHDX-.5D0*DTHDX+THETA                                    SHT05090
      A(IR,J2)= +.5D0*DPHDY-.5D0*DTHDY+THETA                                    SHT05100
      A(IR,JD)= +(DTHDX*DPHDX+DTHDY*DPHDY)/THETA                                SHT05110
     1   -EQP*(2.D0*NE(IC,JC)-NI(IC,JC))                                        SHT05120
     2   +TH(ICM,JC)+TH(ICP,JC)+TH(IC,JCM)+TH(IC,JCP)-4.D0*TH(IC,JC)            SHT05130
     3   -(DTHDX*DTHDX+DTHDY*DTHDY)/THETA                                       SHT05140
     4   -4.D0*THETA                                                            SHT05150
     5   +DT*EQE*THETA*(RECOM*(1.D0-NI(IC,JC))                                  SHT05160
     6      +ALP*(1.D0**2.D0-2.D0*NE(IC,JC)*NI(IC,JC)))                         SHT05170
      A(IR,J4)=-.5D0*DPHDY+.5D0*DTHDY+THETA                                     SHT05180
      A(IR,J5)=-.5D0*DPHDX+.5D0*DTHDX+THETA                                     SHT05190
      A(IR,JD+1)= +EQP*NE(IC,JC)                                                SHT05200
     1   -DT*EQE*THETA*(RECOM*NE(IC,JC)+ALP*NE(IC,JC)*NE(IC,JC))                SHT05210
      C1=DNEDX*DPHDX+DNEDY*DPHDY                                                SHT05220
      C2=-NE(IC,JC)*(DTHDX*DPHDX+DTHDY*DPHDY)/THETA                             SHT05230
CW    C3=NE(IC,JC)*(PH(ICM,JC)+PH(ICP,JC)+PH(IC,JCM)+PH(IC,JCP)                 SHT05240
CW   1       -4.D0*PH(IC,JC))                                                   SHT05250
      C3=NE(IC,JC)*EQP*(NE(IC,JC)-NI(IC,JC))                                    SHT05260
      C4=-(DNEDX*DTHDX+DNEDY*DTHDY)                                             SHT05270
      C5=-NE(IC,JC)*(TH(ICM,JC)+TH(ICP,JC)+TH(IC,JCM)+TH(IC,JCP)                SHT05280
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```
     1       -4.D0*TH(IC,JC))                                                  SHT05290
      C6=NE(IC,JC)*(DTHDX*DTHDX+DTHDY*DTHDY)/THETA                             SHT05300
      C7=-THETA*(NE(ICM,JC)+NE(ICP,JC)+NE(IC,JCM)+NE(IC,JCP)                   SHT05310
     1       -4.D0*NE(IC,JC))                                                  SHT05320
      C8=-DT*EQE*THETA*NEDOT(IC,JC)                                            SHT05330
      C(IR)=C1+C2+C3+C4+C5+C6+C7+C8                                            SHT05340
      GO TO 450                                                                SHT05350
C-----------------------------------------------------------------------       SHT05360
C                                                                              SHT05370
C                   ARRIVE HERE FOR M.EQ.3 (ION CONCEN)                        SHT05380
C                                                                              SHT05390
C-----------------------------------------------------------------------       SHT05400
  430 CONTINUE                                                                 SHT05410
      A(IR,J1)=1.D0-.5D0*BETA*DPHDX                                            SHT05420
      A(IR,J2)=1.D0-.5D0*BETA*DPHDY                                            SHT05430
      A(IR,JD)=-4.D0-BETA*EQP*(2.D0*NI(IC,JC)-NE(IC,JC))                       SHT05440
     1     -DT*EQI*(RECOM*NE(IC,JC)+ALP*NE(IC,JC)*NE(IC,JC))                   SHT05450
      A(IR,J4)=1.D0+.5D0*BETA*DPHDY                                            SHT05460
      A(IR,J5)=1.D0+.5D0*BETA*DPHDX                                            SHT05470
      A(IR,J1-2)=-.5D0*BETA*DNIDX                                              SHT05480
      A(IR,J2-2)=-.5D0*BETA*DNIDY                                              SHT05490
      A(IR,J4-2)=-A(IR,J2-2)                                                   SHT05500
      A(IR,J5-2)=-A(IR,J1-2)                                                   SHT05510
      A(IR,JD-1)=+BETA*EQP*NI(IC,JC)                                           SHT05520
     1       +DT*EQI*(RECOM*(1.D0-NI(IC,JC))                                   SHT05530
     2       +ALP*(1.D0-2.D0*NE(IC,JC)*NI(IC,JC)))                             SHT05540
      C(IR)=-NI(ICP,JC)-NI(ICM,JC)-NI(IC,JCP)-NI(IC,JCM)+4.D0*NI(IC,JC)        SHT05550
     1     -BETA*(DPHDX*DNIDX+DPHDY*DNIDY)                                     SHT05560
     2     -BETA*EQP*(NE(IC,JC)-NI(IC,JC))*NI(IC,JC)                           SHT05570
     3     -EQI*DT*NEDOT(IC,JC)                                                SHT05580
  450 JC=JC+1                                                                  SHT05590
      IF(JC.GT.NS) JC=1                                                        SHT05600
  500 CONTINUE                                                                 SHT05610
C-----------------------------------------------------------------------       SHT05620
C                                                                              SHT05630
C                   SET UP BOUNDARY CONDITIONS (EXCEPT WALL)                   SHT05640
C                                                                              SHT05650
C-----------------------------------------------------------------------       SHT05660
C                                                                              SHT05670
C                   SET UP EQUILIBRIUM B.C.                                    SHT05680
C                                                                              SHT05690
      NXNP=NXNH+M                                                              SHT05700
      DO 530 IR=NXNP,NXN,3                                                     SHT05710
      A(IR,JD)=1.D0                                                            SHT05720
      IF(M.EQ.1) C(IR)=PHQ                                                     SHT05730
      IF(M.EQ.2) C(IR)=NEQ                                                     SHT05740
      IF(M.EQ.3) C(IR)=NIQ                                                     SHT05750
  530 CONTINUE                                                                 SHT05760
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C                                                                       SHT05770
CC                    SET UP NODAL SYMMETRY B.C.                        SHT05780
C                                                                       SHT05790
      IS=NSY3+M                                                         SHT05800
      DO 540 IR=IS,NXNM,NSY3                                            SHT05810
      A(IR,J2)=+0.DO                                                    SHT05820
      A(IR,J4)=+2.DO                                                    SHT05830
  540 CONTINUE                                                          SHT05840
C                                                                       SHT05850
CC                    SET UP INTERNODAL SYMMETRY B.C.                   SHT05860
C                                                                       SHT05870
      IS=2*NSY3-3+M                                                     SHT05880
      DO 550 IR=IS,NXNM,NSY3                                            SHT05890
      A(IR,J4)=+0.DO                                                    SHT05900
      A(IR,J2)=+2.DO                                                    SHT05910
  550 CONTINUE                                                          SHT05920
C                                                                       SHT05930
CC                    NODE CONDITIONS SET                               SHT05940
C                                                                       SHT05950
      A(M,JD)=1.DO                                                      SHT05960
      IF(M.EQ.1) C(1)=PHIB                                              SHT05970
      IF(M.EQ.2) C(M)=NED                                               SHT05980
      IF(M.EQ.3) C(M)=NID                                               SHT05990
 5000 CONTINUE                                                          SHT06000
C-----------------------------------------------------------------------SHT06010
CC                                                                      SHT06020
CC                    MATRIX A IS NOW COMPLETE                          SHT06030
CC                                                                      SHT06040
CC                    COMPUTE SPECTRAL NORM OF  C                       SHT06050
CC                    FOR CONVERGENCE TEST                              SHT06060
CC                                                                      SHT06070
C-----------------------------------------------------------------------SHT06080
      NXM2=NXN-2                                                        SHT06090
      CPH=0.DO                                                          SHT06100
      CNE=0.DO                                                          SHT06110
      CNI=0.DO                                                          SHT06120
      DO 605 K=1,NXM2,3                                                 SHT06130
      K1=K+1                                                            SHT06140
      K2=K+2                                                            SHT06150
      CPH=CPH+C(K)*C(K)                                                 SHT06160
      CNE=CNE+C(K1)*C(K1)                                               SHT06170
  605 CNI=CNI+C(K2)*C(K2)                                               SHT06180
      CER=CPH+CNE+CNI                                                   SHT06190
      WRITE(6,802) ITER,CER,CPH,CNE,CNI,ADJ                             SHT06200
      WRITE(4,802) ITER,CER,CPH,CNE,CNI,ADJ                             SHT06210
      NSSS=NSY3+2                                                       SHT06220
C                                                                       SHT06230
C                                                                       SHT06240
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C                                                                       SHT06250
      CALL LEQT1B(A,NXN,NSSS,NSY3+1,NXN,C,1,NXN,0,XL,IER)               SHT06260
C                                                                       SHT06270
CC                                                                      SHT06280
CC                                                                      SHT06290
CC-----------------------------------------------------------------------SHT06300
CC                                                                      SHT06310
CC                    UPDATE P,NE,NI  FROM VECTOR C                     SHT06320
CC                                                                      SHT06330
C-----------------------------------------------------------------------SHT06340
      CPH=0.D0                                                          SHT06350
      CNE=0.D0                                                          SHT06360
      CNI=0.D0                                                          SHT06370
      CER=0.D0                                                          SHT06380
      DO 7000 M=1,3                                                     SHT06390
      GO TO (710,720,730) ,M                                            SHT06400
  710 CONTINUE                                                          SHT06410
      DO 711 I=1,NC                                                     SHT06420
      K=0                                                               SHT06430
      DO 711 J=1,NSY3,3                                                 SHT06440
      K=K+1                                                             SHT06450
      JJ=J+(I-1)*NSY3                                                   SHT06460
      CPH=CPH+C(JJ)*C(JJ)                                               SHT06470
  711 PH(I,K)=C(JJ)+PH(I,K)                                             SHT06480
      GO TO 7000                                                        SHT06490
  720 CONTINUE                                                          SHT06500
C                                                                       SHT06510
CC                                                                      SHT06520
C                                                                       SHT06530
      DO 721 I=1,NC                                                     SHT06540
      K=0                                                               SHT06550
      DO 721 J=1,NSY3,3                                                 SHT06560
      K=K+1                                                             SHT06570
      JJ=J+(I-1)*NSY3+1                                                 SHT06580
      CNE=CNE+C(JJ)*C(JJ)                                               SHT06590
  721 NE(I,K)=C(JJ)+NE(I,K)                                             SHT06600
      GO TO 7000                                                        SHT06610
  730 CONTINUE                                                          SHT06620
C                                                                       SHT06630
CC                                                                      SHT06640
C                                                                       SHT06650
      DO 731 I=1,NC                                                     SHT06660
      K=0                                                               SHT06670
      DO 731 J=1,NSY3,3                                                 SHT06680
      K=K+1                                                             SHT06690
      JJ=J+(I-1)*NSY3+2                                                 SHT06700
      CNI=CNI+C(JJ)*C(JJ)                                               SHT06710
  731 NI(I,K)=C(JJ)+NI(I,K)                                             SHT06720
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```
 7000 CONTINUE                                                          SHT06730
      CER=CPH+CNE+CNI                                                   SHT06740
      IF(ITER.GT.1) CALL PATCH(NE,0.D0,1.D0)                            SHT06750
      IF(ITER.GT.1) CALL PATCH(NI,0.D0,1.D0)                            SHT06760
      IF(ITER.GT.1) CALL PATCH(PH,0.D0,SPHIB)                           SHT06770
C-----------------------------------------------------------------------SHT06780
C                                                                       SHT06790
C                   CALCULATE THE TEMPERATURE DISTRIBUTION              SHT06800
C                   SEPARATELY FROM THE MATRIX ITERATIVE SCHEME.        SHT06810
C                                                                       SHT06820
C                   COMPUTE AS ISOTHERMAL FOR FIRST ITEMP ITERATIONS    SHT06830
C                                                                       SHT06840
C-----------------------------------------------------------------------SHT06850
      IF(ITER.EQ.ITMAX) GO TO 899                                       SHT06860
      IF(ITER.LT.ITEMP) GO TO 950                                       SHT06870
  899 CONTINUE                                                          SHT06880
      CALL GRDNT(PH,GRDX,GRDY,GRAD)                                     SHT06890
 9876 IF(LADJ.GT.0) ADJ=1.D0/(EQT*(GRAD(NC,1)+GRAD(NC,NS))*.5D0)        SHT06900
      DO 930 J=1,NS                                                     SHT06910
      DO 930 I=1,NC                                                     SHT06920
      TH(I,J)=(12.1D0*DLOG(GRAD(I,J)*EQT*ADJ)+BETA)/BETA                SHT06930
  930 CONTINUE                                                          SHT06940
      CALL PATCH(TH,1.D0,99.D0)                                         SHT06950
      GO TO 990                                                         SHT06960
  950 CONTINUE                                                          SHT06970
      CALL PATCH(TH,1.D0,1.D0)                                          SHT06980
  990 CONTINUE                                                          SHT06990
C-----------------------------------------------------------------------SHT07000
C                                                                       SHT07010
C                   CALCULATE CURRENT MATCHED NED                       SHT07020
C                                                                       SHT07030
C-----------------------------------------------------------------------SHT07040
      WRITE(6,802) ITER,CER,CPH,CNE,CNI,ADJ                             SHT07050
      WRITE(4,802) ITER,CER,CPH,CNE,CNI,ADJ                             SHT07060
C     CALL ANODE(PH,NE,NI,TH,JNODE,JEQUI,TOAD)                          SHT07070
C     NE(I,1)=TOAD                                                      SHT07080
 9000 CONTINUE                                                          SHT07090
      WRITE(6,808)                                                      SHT07100
C-----------------------------------------------------------------------SHT07110
C                                                                       SHT07120
C                   OUTPUT FINAL RESULTS                                SHT07130
C                                                                       SHT07140
C-----------------------------------------------------------------------SHT07150
      CALL OUTPUT(6,PH,UPH,'VOLTAGE POTENTIAL        ')                 SHT07160
      CALL OUTPUT(6,NE,1.D0,'ELECTRON DENSITY        ')                 SHT07170
      CALL OUTPUT(6,NI,1.D0,'ION DENSITY             ')                 SHT07180
      CALL OUTPUT(6,TH,1.D0,'TEMP DISTRIBUTION (ADJ) ')                 SHT07190
      DO 9100 I=1,NC                                                    SHT07200
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      DO 9100 J=1,NS                                                    SHT07210
 9100 GRDX(I,J)=NE(I,J)-NI(I,J)                                         SHT07220
      CALL OUTPUT(6,GRDX,1.D0,'SPACE CHARGE DENSITY    ')               SHT07230
      CALL RTSIDE(NE,NI,TH,NEDOT)                                       SHT07240
      CALL OUTPUT(6,NEDOT,1.D0,'NEDOT DISTRIBUTION      ')              SHT07250
      EFLD=EQT*DENSO*1.D-20                                             SHT07260
      CALL OUTPUT(6,GRAD,EFLD,'ELECTRIC FIELD          ')               SHT07270
      CALL TEMP(PH,TH,ADJ)                                              SHT07280
      CALL OUTPUT(6,TH,1.D0,'REAL TEMP PROFILE (ADJ) ')                 SHT07290
      CALL PATCH(TH,1.D0,99.D0)                                         SHT07300
      CALL JOULE(PH,NE,NI,TH,HEAT)                                      SHT07310
      CALL PATCH(HEAT,1.D0,1.D+10)                                      SHT07320
      DO 9120 I=1,NC                                                    SHT07330
      DO 9120 J=1,NS                                                    SHT07340
      HEAT(I,J)=DLOG10(HEAT(I,J))                                       SHT07350
 9120 CONTINUE                                                          SHT07360
      CALL OUTPUT(6,HEAT,1.D0,'LOG10 JOULE HT (W/M**3) ')               SHT07370
      WRITE(6,805)                                                      SHT07380
      WRITE(6,847) ITMIN,ITMAX,ITNDT,ITEMP,ITBC,LADJ,LPUN,H,FKTR,NAME   SHT07390
      IF(LPUN.EQ.0) WRITE(6,848)                                        SHT07400
      IF(LPUN.GT.0) WRITE(6,849)                                        SHT07410
C                                                                       SHT07420
CC                                                                      SHT07430
C                                                                       SHT07440
      IF(LPUN.EQ.0) GO TO 9300                                          SHT07450
      WRITE(7,800) ITMIN,ITMAX,ITNDT,ITEMP,ITBC,LADJ,LPUN,H,FKTR,NAME   SHT07460
      CALL OUTPUT(7,PH,UPH,'VOLTAGE POTENTIAL      ')                   SHT07470
      CALL OUTPUT(7,NE,1.D0,'ELECTRON DENSITY       ')                  SHT07480
      CALL OUTPUT(7,NI,1.D0,'ION DENSITY            ')                  SHT07490
      CALL TEMP(PH,TH,ADJ)                                              SHT07500
      CALL PATCH(TH,1.D0,99.D0)                                         SHT07510
      CALL OUTPUT(7,TH,1.D0,'TEMP DIST  (TRUNCATED)  ')                 SHT07520
      DO 9200 I=1,NC                                                    SHT07530
      DO 9200 J=1,NS                                                    SHT07540
 9200 GRDX(I,J)=NE(I,J)-NI(I,J)                                         SHT07550
      CALL OUTPUT(7,GRDX,1.D0,'SPACE CHARGE DENSITY    ')               SHT07560
      CALL PATCH(TH,1.D0,99.D0)                                         SHT07570
      CALL RTSIDE(NE,NI,TH,NEDOT)                                       SHT07580
      CALL OUTPUT(7,NEDOT,1.D0,'NEDOT DISTRIBUTION      ')              SHT07590
      CALL GRDNT(PH,GRDX,GRDY,GRAD)                                     SHT07600
      DO 9210 I=1,NC                                                    SHT07610
      DO 9210 J=1,NS                                                    SHT07620
 9210 GRAD(I,J)=DLOG10(GRAD(I,J)*EFLD)                                  SHT07630
      CALL OUTPUT(7,GRAD,1.D0,'LOG10 E-FIELD (V/M)     ')               SHT07640
      CALL OUTPUT(7,HEAT,1.D0,'LOG10 JOULE HT (W/M**3) ')               SHT07650
C                                                                       SHT07660
CC                                                                      SHT07670
C                                                                       SHT07680
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 9300 CONTINUE                                                                  SHT07690
      IF(ITMAX.LE.ITNDT) WRITE(6,850)                                           SHT07700
      IF(ITMAX.GT.ITNDT) WRITE(6,851) ITNDT                                     SHT07710
      IF(ITMAX.GT.ITEMP) WRITE(6,853) ITEMP                                     SHT07720
      IF(ITMAX.LE.ITEMP) WRITE(6,852)                                           SHT07730
      IF(ITMIN.GE.ITBC.OR.ITBC.LE.NBC) WRITE(6,854)                             SHT07740
      IF(ITBC.GT.ITMIN.AND.ITBC.GT.NBC) WRITE(6,855) NBC,ITBC                   SHT07750
      ESP=EQP/HH                                                                SHT07760
      ESE=EQE/HH                                                                SHT07770
      ESI=EQI/HH                                                                SHT07780
      EST=EQT*H                                                                 SHT07790
      WRITE(6,869) ADJ                                                          SHT07800
      WRITE(6,864) BETA,SPHIB,LAMS,ESP,ESE,ESI,EST                              SHT07810
      WRITE(6,865) H,PHIZ,SNED,SNEW,SNID,SNIW,FKTR                              SHT07820
      WRITE(6,866) ALFA1,ALFA2,ALFA3,RECOM                                      SHT07830
      IF(LPUN.LE.0) GO TO 9310                                                  SHT07840
      WRITE(7,869) ADJ                                                          SHT07850
      WRITE(7,864) BETA,SPHIB,LAMS,ESP,ESE,ESI,EST                              SHT07860
      WRITE(7,865) H,PHIZ,SNED,SNEW,SNID,SNIW,FKTR                              SHT07870
      WRITE(7,866) ALFA1,ALFA2,ALFA3,RECOM                                      SHT07880
 9310 CONTINUE                                                                  SHT07890
C-----------------------------------------------------------------------------SHT07900
C                                                                               SHT07910
C                    CALCULATE CURRENT BALANCES                                 SHT07920
C                                                                               SHT07930
C-----------------------------------------------------------------------------SHT07940
      CALL PATCH(TH,1.D0,99.D0)                                                 SHT07950
      CALL JWALL(6,PH,NE,NI,TH)                                                 SHT07960
      CALL JWALL(4,PH,NE,NI,TH)                                                 SHT07970
      IF(LPUN.GT.0) CALL JWALL(7,PH,NE,NI,TH)                                   SHT07980
      CALL ANODE(PH,NE,NI,TH,JNODE,JEQUI,TOAD)                                  SHT07990
      IF(LPUN.GT.0) WRITE(7,867) JNODE,JEQUI                                    SHT08000
      WRITE(6,867) JNODE,JEQUI                                                  SHT08010
      WRITE(4,867) JNODE,JEQUI                                                  SHT08020
      IF(LPUN.GT.0) WRITE(7,868) TOAD                                           SHT08030
      WRITE(6,868) TOAD                                                         SHT08040
      WRITE(4,868) TOAD                                                         SHT08050
      WRITE(6,805)                                                              SHT08060
      GO TO 1                                                                   SHT08070
  800 FORMAT(5(I2,1X),2I1,1X,F5.2,1X,F5.2,1X,10A4)                              SHT08080
  801 FORMAT(3(5X,F5.2))                                                        SHT08090
  802 FORMAT(' (',I2,') ',D10.4,4(5X,D10.4))                                    SHT08100
  803 FORMAT(20A4)                                                              SHT08110
  804 FORMAT(3X,7(1X,D10.4))                                                    SHT08120
  805 FORMAT('1')                                                               SHT08130
  808 FORMAT(' ',13('.....'),/)                                                 SHT08140
  811 FORMAT(' PHD ',F5.2,' PHW ',F5.2,' PHC ',F5.2)                            SHT08150
  812 FORMAT(' NED ',F5.2,' NEW ',F5.2,' NEQ ',F5.2)                            SHT08160
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  813 FORMAT(' NID ',F5.2,' NIW ',F5.2,' NIQ ',F5.2)                              SHT08170
  847 FORMAT(' *CONTROL* ',5(I2,1X),2I1,F5.2,1X,F5.2,1X,10A4)                    SHT08180
  848 FORMAT(/,'      RESULTS NOT PUNCHED')                                      SHT08190
  849 FORMAT(/,'      RESULTS PUNCHED IN D10.4 FORMAT')                          SHT08200
  850 FORMAT(/,'      NEDOT NOT COUPLED')                                        SHT08210
  851 FORMAT(/,'      NEDOT COUPLED AFTER',I3,' ITERATIONS')                     SHT08220
  852 FORMAT(/,'      ISOTHERMAL CONDITIONS THROUGHOUT')                         SHT08230
  853 FORMAT(/,'      THERMAL ROUTINE COUPLED AFTER',I3,' ITERATIONS')           SHT08240
  854 FORMAT(/,'      FLOATING WALL B.C. NOT USED',//)                           SHT08250
  855 FORMAT(/,'      FLOATING WALL B.C. USED FROM',I2,' TO ',I2,//)             SHT08260
  881 FORMAT(/,'      TEMP FIELD IS ADJUSTED FOE EQUI B.C.',//)                  SHT08270
  882 FORMAT(/,'      TEMP FIELD IS NOT ADJUSTED FOR EQUI B.C.',//)              SHT08280
  864 FORMAT(/,21X,'BETA   ',E9.3,/,21X,'PHIB   ',E9.3,/,                        SHT08290
     1      21X,'LAMS   ',E9.3,/,22X,'ESP   ',E9.3,/,                          SHT08300
     2      22X,'ESE   ',E9.3,/,22X,'ESI   ',E9.3,/,22X,'EST   ',E9.3)         SHT08310
  865 FORMAT(/,23X,' H   ',E9.3,/,21X,'PHIZ   ',E9.3,/,                         SHT08320
     1      22X,'NED   ',E9.3,5X,'NEW   ',E9.3,/,                              SHT08330
     2      22X,'NID   ',E9.3,5X,'NIW   ',E9.3,/,20X,'FKTR    ',E9.3)          SHT08340
  866 FORMAT(/,19X,'ALPHA1   ',E9.3,/,19X,'ALPHA2   ',E9.3,/,                   SHT08350
     119X,'ALPHA3   ',E9.3,/,20X,'RECOM   ',E9.3)                               SHT08360
  867 FORMAT(//,20X,'JNODE   ',E10.4,/,18X,' JEQUIL   ',E10.4)                  SHT08370
  868 FORMAT(12X,'J-MATCHED NED   ',E10.4)                                      SHT08380
  869 FORMAT(/,20X,'   ADJ   ',E10.4,/)                                         SHT08390
 9999 STOP                                                                      SHT08400
      END                                                                       SHT08410
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C                                                                               DEL00010
C                                                                               DEL00020
C                                                                               DEL00030
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC DEL00040
C                                                                               DEL00050
C                                                                               DEL00060
C           SUBROUTINE DELOP                                                    DEL00070
C                                                                               DEL00080
C                  REDIMENSIONED                                                DEL00090
C                                                                               DEL00100
C                                                                               DEL00110
C                  DETERMINES THE FIRST ORDER DIFFERENCES OF                    DEL00120
C                  THE SOLUTION SET, (PH,NE,NI,TH), AT THE NODE                 DEL00130
C                  POINT 'IC,JC'.  TWO POINT DIFFERENCES ARE USED.              DEL00140
C                                                                               DEL00150
C                  RETURNS THE DIFFERENCES THROUGH COMMON: DIVER.               DEL00160
C                                                                               DEL00170
C                                                                               DEL00180
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC DEL00190
C                                                                               DEL00200
C                                                                               DEL00210
C                                                                               DEL00220
      SUBROUTINE DELOP(PH,NE,NI,TH)                                             DEL00230
      IMPLICIT REAL*8 (A-H,O-$)                                                 DEL00240
      REAL*8 PH(NCOLS,1),NE(NCOLS,1),NI(NCOLS,1),TH(NCOLS,1)                    DEL00250
      COMMON /IPARM/NCOLS,NSYM,ITER                                             DEL00260
      COMMON /DIVER/ DPHDX,DPHDY,DNEDX,DNEDY,DNIDX,DNIDY,DTHDX,DTHDY            DEL00270
      COMMON /DPARM/ M,IC,JC,ICM,ICP,JCM,JCP                                    DEL00280
      REAL*8 NE,NI,NEDOT                                                        DEL00290
      DPHDX=0.D0                                                                DEL00300
      DPHDY=0.D0                                                                DEL00310
      DNEDX=0.D0                                                                DEL00320
      DNEDY=0.D0                                                                DEL00330
      DNIDX=0.D0                                                                DEL00340
      DNIDY=0.D0                                                                DEL00350
      DTHDX=0.D0                                                                DEL00360
      DTHDY=0.D0                                                                DEL00370
      ICP=IC+1                                                                  DEL00380
      ICM=IC-1                                                                  DEL00390
      JCP=JC+1                                                                  DEL00400
      JCM=JC-1                                                                  DEL00410
      IF(JC.EQ.1) JCM=JCP                                                       DEL00420
      IF(JC.EQ.NSYM) JCP=JCM                                                    DEL00430
      IF(JC.EQ.1) GO TO  310                                                    DEL00440
      IF(JC.EQ.NSYM) GO TO  320                                                 DEL00450
      DPHDX=PH(ICP,JC)-PH(ICM,JC)                                               DEL00460
      DNEDX=NE(ICP,JC)-NE(ICM,JC)                                               DEL00470
      DNIDX=NI(ICP,JC)-NI(ICM,JC)                                               DEL00480
```

```
      DPHDY=PH(IC,JCP)-PH(IC,JCM)                                       DEL00490
      DNEDY=NE(IC,JCP)-NE(IC,JCM)                                       DEL00500
      DNIDY=NI(IC,JCP)-NI(IC,JCM)                                       DEL00510
      DTHDX=TH(ICP,JC)-TH(ICM,JC)                                       DEL00520
      DTHDY=TH(IC,JCP)-TH(IC,JCM)                                       DEL00530
      GO TO  350                                                        DEL00540
  310 IC=IC+1                                                           DEL00550
      ICP=IC+1                                                          DEL00560
      ICM=IC-1                                                          DEL00570
      DPHDX=PH(ICP,1)-PH(ICM,1)                                         DEL00580
      DNEDX=NE(ICP,1)-NE(ICM,1)                                         DEL00590
      DNIDX=NI(ICP,1)-NI(ICM,1)                                         DEL00600
      DTHDX=TH(ICP,1)-TH(ICM,1)                                         DEL00610
      GO TO  350                                                        DEL00620
  320 CONTINUE                                                          DEL00630
      DPHDX=PH(ICP,NSYM)-PH(ICM,NSYM)                                   DEL00640
      DNEDX=NE(ICP,NSYM)-NE(ICM,NSYM)                                   DEL00650
      DNIDX=NI(ICP,NSYM)-NI(ICM,NSYM)                                   DEL00660
      DTHDX=TH(ICP,NSYM)-TH(ICM,NSYM)                                   DEL00670
  350 CONTINUE                                                          DEL00680
      DPHDX=DPHDX/2.D0                                                  DEL00690
      DNEDX=DNEDX/2.D0                                                  DEL00700
      DNIDX=DNIDX/2.D0                                                  DEL00710
      DTHDX=DTHDX/2.D0                                                  DEL00720
      DPHDY=DPHDY/2.D0                                                  DEL00730
      DNEDY=DNEDY/2.D0                                                  DEL00740
      DNIDY=DNIDY/2.D0                                                  DEL00750
      DTHDY=DTHDY/2.D0                                                  DEL00760
      RETURN                                                            DEL00770
      END                                                               DEL00780
```

```
C                                                                       GRD00010
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC GRD00020
C                                                                       GRD00030
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC GRD00040
C                                                                       GRD00050
C                     SUBROUTINE GRDNT                                  GRD00060
C                                                                       GRD00070
C                     COMPUTES THE GRADIENT OF THE INPUTED FIELD        GRD00080
C                     VALUE IN BOTH COORDS AND THE SCALER               GRD00090
C                                                                       GRD00100
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC GRD00110
C                                                                       GRD00120
C                                                                       GRD00130
C                                                                       GRD00140
      SUBROUTINE GRDNT(PH,GX,GY,GRAD)                                   GRD00150
      IMPLICIT REAL*8 (A-H,O-$)                                         GRD00160
      REAL*8 PH(NC,1),GRAD(NC,1),GX(NC,1),GY(NC,1)                      GRD00170
      COMMON /IPARM/ NC,NS,ITER                                         GRD00180
      NCM1=NC-1                                                         GRD00190
      NCM2=NC-2                                                         GRD00200
      DO 900 J=1,NS                                                     GRD00210
      JP=J+1                                                            GRD00220
      JM=J-1                                                            GRD00230
CUT   GX(1,J)=(-3.D0*PH(1,J)+4.D0*PH(2,J)-PH(3,J))/2.D0                 GRD00240
      GX(1,J)=PH(2,J)-PH(1,J)                                           GRD00250
      IF(J.EQ.1) JM=JP                                                  GRD00260
      IF(J.EQ.NS) JP=JM                                                 GRD00270
      GY(1,J)=(PH(1,JP)-PH(1,JM))/2.D0                                  GRD00280
  900 CONTINUE                                                          GRD00290
      DO 910 J=1,NS                                                     GRD00300
      JP=J+1                                                            GRD00310
      JM=J-1                                                            GRD00320
CUT   GX(NC,J)=(-3.D0*PH(NC,J)+4.D0*PH(NCM1,J)-PH(NCM2,J))/2.D0         GRD00330
      GX(NC,J)=PH(NC,J)-PH(NCM1,J)                                      GRD00340
      IF(J.EQ.1) JM=JP                                                  GRD00350
      IF(J.EQ.NS) JP=JM                                                 GRD00360
      GY(NC,J)=(PH(NC,JP)-PH(NC,JM))/2.D0                               GRD00370
  910 CONTINUE                                                          GRD00380
      DO 920 J=1,NS                                                     GRD00390
      DO 920 I=2,NCM1                                                   GRD00400
      IP=I+1                                                            GRD00410
      IM=I-1                                                            GRD00420
      JM=J-1                                                            GRD00430
      JP=J+1                                                            GRD00440
      IF(J.EQ.1) JM=JP                                                  GRD00450
      IF(J.EQ.NS) JP=JM                                                 GRD00460
      GX(I,J)=(PH(IP,J)-PH(IM,J))/2.D0                                  GRD00470
      GY(I,J)=(PH(I,JP)-PH(I,JM))/2.D0                                  GRD00480
```

```
920 CONTINUE                                                          GRD00490
    DO 950 I=1,NC                                                     GRD00500
    DO 950 J=1,NS                                                     GRD00510
950 GRAD(I,J)=(GX(I,J)*GX(I,J)+GY(I,J)*GY(I,J))**.5D0                 GRD00520
    RETURN                                                            GRD00530
    END                                                               GRD00540
```

```
C                                                                       TEM00010
CC                                                                      TEM00020
CCC                                                                     TEM00030
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCTEM00040
C                                                                       TEM00050
C                   SUBROUTINE TEMP                                     TEM00060
C                                                                       TEM00070
C                   CALCULATES THE ELECTRON TEMPERATURE DISTRIBUTION    TEM00080
C                   GIVEN THE VOLTAGE POTENTIAL GRADIENTS.              TEM00090
C                                                                       TEM00100
C                                                                       TEM00110
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCTEM00120
C                                                                       TEM00130
C                                                                       TEM00140
C                                                                       TEM00150
      SUBROUTINE TEMP(PH,TH,ADJ)                                        TEM00160
      IMPLICIT REAL*8(A-H,O-$)                                          TEM00170
      EXTERNAL GRDNT                                                    TEM00180
      REAL*8 PH(NC,1),TH(NC,1),GRAD(25,7),GRDX(25,7),GRDY(25,7)         TEM00190
      REAL*8 LAMS                                                       TEM00200
      COMMON /IPARM/ NC,NS,IT                                           TEM00210
      COMMON /CURT/ DEO,DIO,LAMS,BETA                                   TEM00220
      COMMON /EQUATS/ CE,CJ,EQP,EQE,EQI,EQT                             TEM00230
      CALL GRDNT(PH,GRDX,GRDY,GRAD)                                     TEM00240
      DO 100 I=1,NC                                                     TEM00250
      DO 100 J=1,NS                                                     TEM00260
  100 TH(I,J)=(12.1D0*DLOG(GRAD(I,J)*EQT*ADJ)+BETA)/BETA                TEM00270
      RETURN                                                            TEM00280
      END                                                               TEM00290
```

```
C                                                                               RTS00010
C                                                                               RTS00020
C                                                                               RTS00030
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCRTS00040
C                                                                               RTS00050
C     SUBROUTINE RTSIDE                                                         RTS00060
C                                                                               RTS00070
C        USED TO CALCULATE THE NEDOT TERM OVER THE FIELD                        RTS00080
C                2 BODY                                                         RTS00090
C                3 BODY (HEAVY)                                                 RTS00100
C                3 BODY (ELECT)..HIN & HIRSCH                                   RTS00110
C                                                                               RTS00120
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCRTS00130
C                                                                               RTS00140
C                                                                               RTS00150
C                                                                               RTS00160
      SUBROUTINE RTSIDE(NE,NI,TH,NEDOT)                                         RTS00170
      IMPLICIT REAL*8 (A-H,O-$)                                                 RTS00180
      REAL*8 NE(NCOLS,1),NI(NCOLS,1),TH(NCOLS,1),NEDOT(NCOLS,1)                 RTS00190
      COMMON /IPARM/ NCOLS,NSYM,ITER                                            RTS00200
      COMMON /IONIZ/ RECOM,A1,A2,A3,T                                           RTS00210
      DO 10 J=1,NSYM                                                            RTS00220
      DO 10 I=1,NCOLS                                                           RTS00230
   10 NEDOT(I,J)=NE(I,J)*(1.D0-NI(I,J))*RECOM                                   RTS00240
     1    +A2*(TH(I,J)*T)**(-4.5D0)*(1.D0-NE(I,J)*NI(I,J))*NE(I,J)              RTS00250
      RETURN                                                                    RTS00260
      END                                                                       RTS00270
```

```
C                                                                               PAT00010
C                                                                               PAT00020
C                                                                               PAT00030
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCPAT00040
C                                                                               PAT00050
C                                                                               PAT00060
C                     SUBROUTINE PATCH                                          PAT00070
C                                                                               PAT00080
C                     BOUNDS THE ARRAY Q(I,J)                                   PAT00090
C                     WITHIN THE LOWER AND UPPER BOUNDS                         PAT00100
C                     (XL,XU RESPECTIVELY).                                     PAT00110
C                                                                               PAT00120
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCPAT00130
C                                                                               PAT00140
C                                                                               PAT00150
C                                                                               PAT00160
      SUBROUTINE PATCH(Q,XL,XU)                                                 PAT00170
      REAL*8 Q(NC,1),XU,XL                                                      PAT00180
      COMMON /IPARM/ NC,NS,IT                                                   PAT00190
      DO 100 I=1,NC                                                             PAT00200
      DO 100 J=1,NS                                                             PAT00210
      IF(Q(I,J).GT.XU) Q(I,J)=XU                                                PAT00220
      IF(Q(I,J).LT.XL) Q(I,J)=XL                                                PAT00230
  100 CONTINUE                                                                  PAT00240
      RETURN                                                                    PAT00250
      END                                                                       PAT00260
```

```
C                                                                               AN000010
CCC                                                                             AN000020
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC AN000030
CCC                                                                             AN000040
CCC           SUBROUTINE ANODE                                                  AN000050
CCC                                                                             AN000060
CCC                CALCULATES JEQUI AND JNODE                                   AN000070
CCC                CALCULATES NE(ANODE) REQUIRED FOR JEQUI=JNODE                AN000080
CCC                                                                             AN000090
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC AN000100
CCC                                                                             AN000110
C                                                                               AN000120
C                                                                               AN000130
C                                                                               AN000140
      SUBROUTINE ANODE (PH,NE,NI,TH,JNODE,JEQUI,NED)                            AN000150
      IMPLICIT REAL*8(A-H,O-$)                                                  AN000160
      COMMON /IPARM/ NC,NS,IT                                                   AN000170
      COMMON /CURT/ DEO,DIO,LAMS,BETA                                           AN000180
      COMMON /EQUATS/ CE,CJ,EQP,EQE,EQI,EQT                                     AN000190
      REAL*8 LAMS,NED,JCALC,JEQUI,JNSAV,JNODE,JX,JY                             AN000200
      REAL*8 PH(NC,1),NE(NC,1),NI(NC,1),TH(NC,1)                                AN000210
C                                                                               AN000220
CCC           JNODE,JEQUI ARE IN MKS                                            AN000230
C                                                                               AN000240
      NED=NE(1,1)                                                               AN000250
CZ    DPHDX=(-3.D0*PH(1,1)+4.D0*PH(2,1)-PH(3,1))/2.D0                           AN000260
CZ    DNEDX=(-3.D0*NE(1,1)+4.D0*NE(2,1)-NE(3,1))/2.D0                           AN000270
CZ    DPHDY=(-3.D0*PH(1,1)+4.D0*PH(1,2)-PH(1,3))/2.D0                           AN000280
CZ    DNEDY=(-3.D0*NE(1,1)+4.D0*NE(1,2)-NE(1,3))/2.D0                           AN000290
      JEQUI=.5D0*(-NE(NC,1)*(PH(NC,1)-PH(NC-1,1))/TH(NC,1)                      AN000300
     1  +(NE(NC,1)-NE(NC-1,1)))                                                 AN000310
      JEQUI=JEQUI+.5D0*(-NE(NC,NS)*(PH(NC,NS)-PH(NC-1,NS))/TH(NC,NS)            AN000320
     1  +(NE(NC,NS)-NE(NC-1,NS)))                                               AN000330
      NSLL=NS-1                                                                 AN000340
      DO  100 J=2,NSLL                                                          AN000350
      JEQUI=JEQUI-NE(NC,J)*(PH(NC,J)-PH(NC-1,J))/TH(NC,J)                       AN000360
     1  +(NE(NC,J)-NE(NC-1,J))                                                  AN000370
  100 CONTINUE                                                                  AN000380
CZ    DZEDX=(-3.D0*NED+4.D0*NE(2,1)-NE(3,1))/2.D0                               AN000390
CZ    DZEDY=(-3.D0*NED+4.D0*NE(1,2)-NE(1,3))/2.D0                               AN000400
      DPHDX=PH(2,1)-PH(1,1)                                                     AN000410
      DNEDX=NE(2,1)-NE(1,1)                                                     AN000420
      DPHDY=PH(1,2)-PH(1,1)                                                     AN000430
      DNEDY=NE(1,2)-NE(1,1)                                                     AN000440
      JX=.5D0*(-NED*DPHDX/TH(1,1)+DNEDX)                                        AN000450
      JY=     (-NED*DPHDY/TH(1,1)+DNEDY)                                        AN000460
      JNODE=(JX*JX+JY*JY)**.5D0                                                 AN000470
      JCALC=JNODE                                                               AN000480
```

```
      DEL=-.01D0                                                        AN000490
  150 CONTINUE                                                          AN000500
      NED=NED+DEL                                                       AN000510
      DZEDX=NE(2,1)-NED                                                 AN000520
      DZEDY=NE(1,2)-NED                                                 AN000530
      JNSAV=JCALC                                                       AN000540
      JX=.5D0*(-NED*DPHDX/TH(1,1)+DZEDX)                                AN000550
      JY=      (-NED*DPHDY/TH(1,1)+DZEDY)                               AN000560
      JCALC=(JX*JX+JY*JY)**.5D0                                         AN000570
      IF(JCALC.GT.JEQUI.AND.JCALC.LE.JNSAV) GO TO 150                   AN000580
      IF(JCALC.LE.JEQUI.AND.JCALC.GE.JNSAV) GO TO 150                   AN000590
      DEL=-DEL*.1D0                                                     AN000600
      IF(DABS(DEL).GE..0001D0) GO TO 150                                AN000610
      JNODE=JNODE*CJ*DEQ                                                AN000620
      JEQUI=JEQUI*CJ*DEQ                                                AN000630
      RETURN                                                            AN000640
      END                                                               AN000650
```

```
C                                                                               JWA00010
C                                                                               JWA00020
C                                                                               JWA00030
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC JWA00040
C                                                                               JWA00050
C                     SUBROUTINE JWALL                                          JWA00060
C                                                                               JWA00070
C                                                                               JWA00080
C                     CALCULATES AND PRINTS OUT THE                             JWA00090
C                     CURRENT TO THE WALL DUE TO ELECTRONS AND IONS             JWA00100
C                                                                               JWA00110
C                                                                               JWA00120
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC JWA00130
C                                                                               JWA00140
C                                                                               JWA00150
C                                                                               JWA00160
      SUBROUTINE JWALL(IOP,PH,NE,NI,TH)                                         JWA00170
      IMPLICIT REAL *8 (A-H,O-$)                                                JWA00180
      REAL*8 LAMS,JX(7),J2(7)                                                   JWA00190
      REAL*8 PH(NC,1),NE(NC,1),NI(NC,1),TH(NC,1)                                JWA00200
      COMMON /CURT/ DEO,DIO,LAMS,BETA                                           JWA00210
      COMMON /EQUATS/ CE,CJ,EQP,EQE,EQI,EQT                                     JWA00220
      COMMON /IPARM/ NC,NS,IT                                                   JWA00230
      DO 10 J=1,NS                                                              JWA00240
      T=TH(1,J)                                                                 JWA00250
      DP2=PH(2,J)-PH(1,J)                                                       JWA00260
      DN2=NE(2,J)-NE(1,J)                                                       JWA00270
      DI2=NI(2,J)-NI(1,J)                                                       JWA00280
      JX(J)=CJ*(DEO*(DP2*NE(1,J)/T-DN2)+DIO*(BETA*NI(1,J)*DP2+DI2))             JWA00290
      J2(J)=CJ*(DEO*(DP2*NE(1,J)/T-DN2))                                        JWA00300
   10 CONTINUE                                                                  JWA00310
      WRITE(IOP,821)                                                            JWA00320
      WRITE(IOP,819)                                                            JWA00330
      WRITE(IOP,818) (NODE,NODE=1,NS)                                           JWA00340
      WRITE(IOP,820) (JX(J),J=1,NS)                                             JWA00350
      WRITE(IOP,822) (J2(J),J=1,NS)                                             JWA00360
      WRITE(IOP,821)                                                            JWA00370
  818 FORMAT('       ',7('    (',I1,')    '),/)                                 JWA00380
  819 FORMAT('  CURRENT FLOW PERPENDICULAR TO WALL AT EACH NODE',/               JWA00390
     $       '  JX3=J(ELECT+ION)     JX2=J(ELECT)',/)                          JWA00400
  820 FORMAT(' JX3 ',7(1X,D9.3))                                                JWA00410
  822 FORMAT(' JX2 ',7(1X,D9.3))                                                JWA00420
  821 FORMAT(' ',13('......'),/)                                                JWA00430
      RETURN                                                                    JWA00440
      END                                                                       JWA00450
```

```
C                                                                       JOU00010
C                                                                       JOU00020
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCJOU00030
C                                                                       JOU00040
C                   SUBROUTINE JOULE                                    JOU00050
C                                                                       JOU00060
C                   CALCULATES THE JOULE HEAT DISTRIBUTION              JOU00070
C                   (ARRAY-HEAT).                                       JOU00080
C                                                                       JOU00090
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCJOU00100
C                                                                       JOU00110
C                                                                       JOU00120
C                                                                       JOU00130
      SUBROUTINE JOULE(PH,NE,NI,TH,HEAT)                                JOU00140
      IMPLICIT REAL*8 (A-H,O-$)                                         JOU00150
      EXTERNAL GRDNT,OUTPUT                                             JOU00160
      REAL*8 LAMS,JX,JY,GRAD(25,7),DPX(25,7),DPY(25,7)                  JOU00170
      REAL*8 DEX(25,7),DEY(25,7),DIX(25,7),DIY(25,7)                    JOU00180
      REAL*8 PH(NC,1),NE(NC,1),NI(NC,1),TH(NC,1),HEAT(NC,1)             JOU00190
      COMMON /CURT/ DEO,DIO,LAMS,B                                      JOU00200
      COMMON /IONIZ/ REC,A1,A2,A3,TSTAR                                 JOU00210
      COMMON /IPARM/ NC,NS,IT                                           JOU00220
      COMMON /EQUATS/ CE,CJ,EQP,EQE,EQI,EQT                             JOU00230
C                                                                       JOU00240
CC          HEAT IS IN MKS UNITS                                        JOU00250
C                                                                       JOU00260
      COEF=CE*CJ                                                        JOU00270
      CALL GRDNT(PH,DPX,DPY,GRAD)                                       JOU00280
      CALL GRDNT(NE,DEX,DEY,GRAD)                                       JOU00290
      CALL GRDNT(NI,DIX,DIY,GRAD)                                       JOU00300
      DO 100 J=1,NS                                                     JOU00310
      DO 100 I=1,NC                                                     JOU00320
      JX=DEO*(DPX(I,J)*NE(I,J)/TH(I,J)-DEX(I,J))                        JOU00330
      JY=DEO*(DPY(I,J)*NE(I,J)/TH(I,J)-DEY(I,J))                        JOU00340
      DEX(I,J)=JX                                                       JOU00350
      DEY(I,J)=JY                                                       JOU00360
      F1=JX*DPX(I,J)                                                    JOU00370
      F2=JY*DPY(I,J)                                                    JOU00380
      HEAT(I,J)=COEF*(F1*F1+F2*F2)**.5D0                                JOU00390
  100 CONTINUE                                                          JOU00400
      CALL OUTPUT(7,DEX,CJ,' J DENSITY-X (AMP/M**2) ')                  JOU00410
      CALL OUTPUT(7,DEY,CJ,' J DENSITY-Y (AMP/M**2) ')                  JOU00420
      RETURN                                                            JOU00430
      END                                                               JOU00440
```

```
C                                                                       OUT00010
C                                                                       OUT00020
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC OUT00030
C                                                                       OUT00040
C                                                                       OUT00050
C               SUBROUTINE OUTPUT                                       OUT00060
C                                                                       OUT00070
C               OUTPUTS TO THE DESIGNATED DEVICE..IOD                   OUT00080
C               THE Q-ARRAY PASSED AND LABELED                          OUT00090
C               WITH THE LIST PASSED IN THE ARGUMENTS.                  OUT00100
C                                                                       OUT00110
C                                                                       OUT00120
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC OUT00130
C                                                                       OUT00140
C                                                                       OUT00150
C                                                                       OUT00160
      SUBROUTINE OUTPUT (IOD,Q,COF,LIST)                                OUT00170
      IMPLICIT REAL*8 (A-H,O-$)                                         OUT00180
      COMMON /IPARM/ NC,NS,IT                                           OUT00190
      REAL*4 LIST(6)                                                    OUT00200
      REAL*8 Q(NC,1),T(7)                                               OUT00210
      WRITE(IOD,800) IT,(LIST(I),I=1,6)                                 OUT00220
      DO 10 I=1,NC                                                      OUT00230
      DO 11 J=1,NS                                                      OUT00240
   11 T(J)=Q(I,J)*COF                                                   OUT00250
      WRITE(IOD,801)I,(T(J),J=1,NS)                                     OUT00260
   10 CONTINUE                                                          OUT00270
  800 FORMAT(' (',I2,')          ',6A4)                                 OUT00280
  801 FORMAT(' ',I2,7(1X,D10.4))                                        OUT00290
      RETURN                                                            OUT00300
      END                                                               OUT00310
```

```
C                                                                                PL000010
C                                                                                PL000020
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCPL000030
C                                                                                PL000040
C                                                                                PL000050
C                                                                                PL000060
C        PROGRAM PLOT                                                            PL000070
C                                                                                PL000080
C            THIS PROGRAM CONSTRUCTS A PLOT OF                                   PL000090
C               (1) POTENTIAL VS. DISTANCE FROM WALL                             PL000100
C               (2) NE AND NI DENSITIES VS. DISTANCE FROM WALL                   PL000110
C               (3) SPACE CHARGE VS. DISTANCE FROM THE WALL                      PL000120
C               (4) ELECTRIC FIELD (V/M)                                         PL000130
C                                                                                PL000140
C            THE PLOTS ARE OF VALUES TAKEN ALONG THE ANODE LINE                  PL000150
C            OF SYMMETRY.  INPUT DATA CONSIST OF THE OUTPUT                      PL000160
C            FROM THE SHEATH PROGRAM FORMATED AS E10.4                           PL000170
C                                                                                PL000180
C            CHANGE FORMAT 803 FOR DESIRED CUTS                                  PL000190
C               "NX", WHERE N=4+11*COL                                           PL000200
C                                                                                PL000210
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCPL000220
C                                                                                PL000230
C                                                                                PL000240
C                                                                                PL000250
      REAL*4 LX(5)/'DIST','ANCE',' (MM',')  ','    '/                           PL000260
      REAL*4 LYN(5)/'SPEC','IES ','DENS','ITY ','    '/                         PL000270
      REAL*4 LYS(5),LYP(5),LYX(5),LYH(5)                                         PL000280
      REAL*4 X(25),YP(25),YE(25),YI(25),YT(25),YS(25),YD(25),YX(25)              PL000290
      REAL*4 YH(25)                                                              PL000300
      REAL*4 ALP(10)                                                             PL000310
      DIMENSION K1(6)                                                            PL000320
      NC=25                                                                      PL000330
      XL=4.5                                                                     PL000340
      YL=3.75                                                                    PL000350
    1 CONTINUE                                                                   PL000360
      READ(5,800) (K1(I),I=1,6),HH,ST,ALP                                        PL000370
CUT   IF(K1(1).LE.0) GO TO 900                                                   PL000380
      READ(5,802) LYP                                                            PL000390
      READ(5,803) (YP(I),I=1,NC)                                                 PL000400
      READ(5,802) DUM                                                            PL000410
      READ(5,803) (YE(I),I=1,NC)                                                 PL000420
      READ(5,802) DUM                                                            PL000430
      READ(5,803) (YI(I),I=1,NC)                                                 PL000440
      READ(5,802) DUM                                                            PL000450
      READ(5,803) (YT(I),I=1,NC)                                                 PL000460
      READ(5,802) LYS                                                            PL000470
      READ(5,803) (YS(I),I=1,NC)                                                 PL000480
```

```
      READ(5,802) DUM                                                          PL000490
      READ(5,803) (YD(I),I=1,NC)                                               PL000500
      READ(5,802) LYX                                                          PL000510
      READ(5,803) (YX(I),I=1,NC)                                               PL000520
      READ(5,802) LYH                                                          PL000530
      READ(5,803) (YH(I),I=1,NC)                                               PL000540
C                  DO 17 I=1,25                                                PL000550
C  17              YH(I)=ALOG10(YH(I))                                         PL000560
      WRITE(6,806)K1,HH,ST,ALP                                                 PL000570
      WRITE(6,805)                                                             PL000580
      DO 20 K=1,NC                                                             PL000590
   20 WRITE(6,808) K,YP(K),YE(K),YI(K),YT(K),YS(K),YD(K),YX(K)                 PL000600
      YSMIN=0.0                                                                PL000610
      YSMAX=0.0                                                                PL000620
      YXMAX=0.0                                                                PL000630
      YHMAX=0.0                                                                PL000640
      DO 40 I=1,NC                                                             PL000650
      IF(YSMIN.GT.YS(I)) YSMIN=YS(I)                                           PL000660
      IF(YSMAX.LT.YS(I)) YSMAX=YS(I)                                           PL000670
      IF(YXMAX.LT.YX(I)) YXMAX=YX(I)                                           PL000680
      IF(YHMAX.LT.YH(I)) YHMAX=YH(I)                                           PL000690
   40 CONTINUE                                                                 PL000700
      YSMAX=2.*YSMAX                                                           PL000710
      YSMIN=YSMIN*2.                                                           PL000720
C                                                                              PL000730
      YPZERO=35.                                                               PL000740
C                                                                              PL000750
      DO 50 I=1,NC                                                             PL000760
   50 X(I)=FLOAT(I-1)*HH*7.434273E-03*YPZERO**.5                               PL000770
      CALL PLOTG(X,YP,NC,1,1,0,LX,20,LYP,20,0.,X(NC),0.,YPZERO,XL,YL)          PL000780
      CALL PLOTG(X,YE,NC,1,1,0,LX,20,LYN,20,0.,X(NC),0.,1.,XL,YL)              PL000790
      CALL PLOTG(X,YI,NC,2,1,0,LX,20,LYN,20,0.,X(NC),0.,1.,XL,YL)              PL000800
      CALL PLOTG(X,YS,NC,1,1,0,LX,20,LYS,20,0.,X(NC),YSMIN,YSMAX,XL,YL)        PL000810
      CALL PLOTG(X,YX,NC,1,1,0,LX,20,LYX,20,0.,X(NC),0.,YXMAX,XL,YL)           PL000820
      CALL PLOTG(X,YH,NC,1,1,0,LX,20,LYH,20,0.,X(NC),0.,YHMAX,XL,YL)           PL000830
      CALL PLOT(0.,0.,999)                                                     PL000840
  800 FORMAT(6(I2,1X),F5.2,1X,F5.2,1X,10A4)                                    PL000850
  802 FORMAT(12X,6A4)                                                          PL000860
  803 FORMAT(4X,D10.4)                                                         PL000870
  805 FORMAT(///,' ',10X,'PH',10X,'NE',10X,'NI',10X,'TH',7X,'SPC CHRG',       PL000880
     1       5X,'NEDOT',7X,'EFLD',/)                                           PL000890
  806 FORMAT(' ** LEAD CARD ** ',6(I2,1X),2(F5.2,1X),10A4)                     PL000900
  808 FORMAT(' (',I2,')',1(2X,E10.4))                                          PL000910
CUT/  GO TO 1                                                                  PL000920
  900 CONTINUE                                                                 PL000930
      STOP                                                                     PL000940
      END                                                                      PL000950
```

```
C                                                                       PIC00010
C                                                                       PIC00020
C                                                                       PIC00030
C     ****************************************************************  PIC00040
C     *                                                              *  PIC00050
C     *                                                              *  PIC00060
C     *   PROGRAM PICT                                               *  PIC00070
C     *                                                              *  PIC00080
C     *       PROGRAM TO DISPLAY THE PUNCHED OUTPUT FROM             *  PIC00090
C     *       PROGRAM SHEATH IN THREE DIMENSIONAL PRESENTATIONS.     *  PIC00100
C     *                                                              *  PIC00110
C     *       DISPLAYS ARE OF THE FORM Z=F(X,Y), WHERE Z IS...       *  PIC00120
C     *         (1) VOLTAGE POTENTIAL DISTRIBUTION                   *  PIC00130
C     *         (2) NE DENSITY DISTRIBUTION                          *  PIC00140
C     *         (3) NI DENSITY DISTRIBUTION                          *  PIC00150
C     *         (4) TEMPERATURE DISTRIBUTION                         *  PIC00160
C     *         (5) SPACE CHARGE DISTRIBUTION                        *  PIC00170
C     *         (6) NEDOT DISTRIBUTION                               *  PIC00180
C     *         (7) LOG10 E-FIELD                                    *  PIC00190
C     *         (8) JOULE HEATING DISTRIBUTION                       *  PIC00200
C     *                                                              *  PIC00210
C     *       THREE DIMENSIONAL SHEATH DISPLAY                       *  PIC00220
C     *                                                              *  PIC00230
C     *                                                              *  PIC00240
C     ****************************************************************  PIC00250
C                                                                       PIC00260
C                                                                       PIC00270
      EXTERNAL PLOT,PLT3D1,PATCH                                        PIC00280
      REAL*8 TTL(12)                                                    PIC00290
      REAL*4 D(25,7),X(25),Y(19),F(2),SZ(2)                             PIC00300
      REAL*4 DD(25,19),WK(25,19,3)                                      PIC00310
      INTEGER*4 KX(400),KY(400)                                         PIC00320
      INTEGER*4 KONT(8)                                                 PIC00330
      DATA KONT/8*1/                                                    PIC00340
      LOGICAL*1 IDN(25,19)                                              PIC00350
      COMMON /IPARM/ NC,NS                                              PIC00360
      NC=25                                                             PIC00370
      XNC=FLOAT(NC)                                                     PIC00380
      NS=7                                                              PIC00390
      LAST=8                                                            PIC00400
      DO 8 I=1,NC                                                       PIC00410
    8 X(I)=FLOAT(I)/XNC                                                 PIC00420
      NOFOLD=2                                                          PIC00430
      JM=NS*NOFOLD-NOFOLD+1                                             PIC00440
      DO 9 J=1,JM                                                       PIC00450
    9 Y(J)=FLOAT(J)/XNC                                                 PIC00460
      READ(5,802) (KONT(I),I=1,8)                                       PIC00470
                                                                        PIC00480
```

```
      READ(5,800) (TTL(KINK),KINK=7,12)                                  PIC00490
      DO 1000 KORK=1,LAST                                                PIC00500
      READ(5,800) (TTL(KINK),KINK=1,6)                                   PIC00510
      DO 10 I=1,NC                                                       PIC00520
      IK=I                                                               PIC00530
      IF(KORK.EQ.2.OR.KORK.EQ.3) IK=NC-I+1                               PIC00540
   10 READ(5,801) (D(IK,J),J=1,NS)                                       PIC00550
CUT   IF(KORK.EQ.4) CALL PATCH(D,1.,99.)                                 PIC00560
      IF(KONT(KORK).GT.0) WRITE(6,806)                                   PIC00570
      IF(KONT(KORK).LE.0) WRITE(6,807)                                   PIC00580
      WRITE(6,808) (TTL(KINK),KINK=1,6),(NORK,NORK=1,NS)                 PIC00590
      IF(KONT(KORK).LE.0) GO TO 1000                                     PIC00600
      XMAX=0.0                                                           PIC00610
      DO 11 I=1,NC                                                       PIC00620
      DO 11 J=1,NS                                                       PIC00630
      IF(XMAX.LE.D(I,J)) XMAX=D(I,J)                                     PIC00640
   11 CONTINUE                                                           PIC00650
      DO 12 I=1,NC                                                       PIC00660
      DO 12 J=1,NS                                                       PIC00670
      D(I,J)=D(I,J)/XMAX                                                 PIC00680
      DD(I,J+6)=D(I,J)                                                   PIC00690
      DD(I,8-J)=D(I,J)                                                   PIC00700
      DD(I,20-J)=D(I,J)                                                  PIC00710
   12 CONTINUE                                                           PIC00720
      DO 20 I=1,NC                                                       PIC00730
      WRITE(6,809) I,(D(I,J),J=1,NS)                                     PIC00740
   20 CONTINUE                                                           PIC00750
C                                                                        PIC00760
CCC   ********************************************                       PIC00770
CCC   *     BETA ROTATES FIRST ABOUT Z AXIS       *                      PIC00780
CCC   *     ALPHA ROTATES SECOND AROUND Y AXIS    *                      PIC00790
CCC   ********************************************                       PIC00800
C                                                                        PIC00810
      BETA=30.                                                           PIC00820
      ALPHA=30.                                                          PIC00830
      F(1)=1.E+06                                                        PIC00840
      F(2)=F(1)                                                          PIC00850
      SZ(1)=2.                                                           PIC00860
      SZ(2)=SZ(1)                                                        PIC00870
      NK=400                                                             PIC00880
      LNS=2                                                              PIC00890
      CALL PLT3D1(X,NC,Y,JM,DD,ALPHA,BETA,F,TTL,SZ,WK,IDN,KX,KY,NK,LNS)  PIC00900
 1000 CONTINUE                                                           PIC00910
  800 FORMAT(6A8)                                                        PIC00920
  801 FORMAT(3X,7(1X,E10.4))                                             PIC00930
  802 FORMAT(8I1)                                                        PIC00940
  806 FORMAT('1    *** PLOTTED ***',/)                                   PIC00950
  807 FORMAT('1    *** NOT PLOTTED ***',/)                               PIC00960
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  808 FORMAT('0',6A8,//,'     ',7(9X,'(',I2,')'),/)                    PIC00970
  809 FORMAT('     (',I2,')',7(3X,E10.4))                               PIC00980
      STOP 1                                                           PIC00990
      END                                                              PIC01000
C                                                                      PIC01010
CCC                                                                    PIC01020
C                                                                      PIC01030
      SUBROUTINE PATCH(Q,XL,XU)                                        PIC01040
      REAL*4 Q(NC,1),XU,XL                                             PIC01050
      COMMON /IPARM/ NC,NS                                             PIC01060
      DO 100 I=1,NC                                                    PIC01070
      DO 100 J=1,NS                                                    PIC01080
      IF(Q(I,J).GT.XU) Q(I,J)=XU                                       PIC01090
      IF(Q(I,J).LT.XL) Q(I,J)=XL                                       PIC01100
  100 CONTINUE                                                         PIC01110
      RETURN                                                           PIC01120
      END                                                              PIC01130
```

```
C                                                                       STR00010
C                                                                       STR00020
C          ************************************                         STR00030
C          *                                  *                         STR00040
C          *      CURRENT STREAM PROFILE      *                         STR00050
C          *                                  *                         STR00060
C          ************************************                         STR00070
C                                                                       STR00080
C  MAY 1981                                                             STR00090
C                                                                       STR00100
C  PROGRAM TO GENERATE CURRENT STREAMLINES ON THE VERSATEC PLOTTER       STR00110
C  FROM TWO DATA ARRAYS, WHICH CONTAIN THE X,Y MAGNITUDE COMPONENTS.     STR00120
C                                                                       STR00130
C  A VECTOR FIELD IS GENERATED AND USING A 'NEAREST NEIGHBOR'           STR00140
C  PRINCIPLE (4 NEIGHBORS), A LOCAL DIRECTION IS DETERMINED,            STR00150
C  WHICH IS SUCCESSIVELY USED TO 'MARCH OUT' A STREAMLINE.              STR00160
C                                                                       STR00170
C  INPUT:   CONSISTS OF THE FIRST TWO SECTIONS OF PUNCHED OUTPUT        STR00180
C           FROM PROGRAM 'SHTH.'                                        STR00190
C                                                                       STR00200
C  OUTPUT:  PRINTOUT OF INPUTED ARRAYS,                                 STR00210
C           MARCHING PATH OF A STREAMLINE                               STR00220
C           ONE VERSATEC PLOT.                                          STR00230
C                                                                       STR00240
C                                                                       STR00250
C                                                                       STR00260
      REAL*4 JX(25,7),JY(25,7),DUMMY(20),X(25),Y(7),XP(300),YP(300)     STR00270
      REAL*4 VX(25,7),VY(25,7),XS(6),YS(6)                              STR00280
      INTEGER INT(6)                                                    STR00290
      COMMON / SET/ VX,VY,X,Y                                           STR00300
      UNIT=.40                                                          STR00310
      NC=25                                                             STR00320
      NS=7                                                              STR00330
      READ(5,800) INT,H,(DUMMY(JKL),JKL=1,12)                           STR00340
      WRITE(6,810) INT,H,(DUMMY(JKL),JKL=1,12)                          STR00350
        DV=H*7.434273E-03*35.**.5                                       STR00360
      READ(5,801) DUMMY                                                 STR00370
      WRITE(6,801) DUMMY                                                STR00380
C                                                                       STR00390
C        SET UP STREAM STARTING COORDINATES                             STR00400
C                                                                       STR00410
      NSM1=NS-1                                                         STR00420
      DO 8 J=1,NSM1                                                     STR00430
      YS(J)=FLOAT(J-1)+.5                                               STR00440
8     XS(J)=FLOAT(NC-1)*.5                                              STR00450
      DO 10 I=1,NC                                                      STR00460
      READ(5,802) (JX(I,J),J=1,NS)                                      STR00470
      WRITE(6,802) (JX(I,J),J=1,NS)                                     STR00480
```

```
   10     CONTINUE                                                        STR00490
          READ(5,801) DUMMY                                               STR00500
          WRITE(6,801) DUMMY                                              STR00510
          DO 12 I=1,NC                                                    STR00520
          READ(5,802) (JY(I,J),J=1,NS)                                    STR00530
          WRITE(6,802) (JY(I,J),J=1,NS)                                   STR00540
   12     CONTINUE                                                        STR00550
C                                                                         STR00560
CC                    COMPUTE VX AND VY                                   STR00570
C                                                                         STR00580
               DO 14 I=1,NC                                               STR00590
               DO 14 J=1,NS                                               STR00600
               HYP=(JX(I,J)*JX(I,J)+JY(I,J)*JY(I,J))**.5                  STR00610
               VX(I,J)=JX(I,J)/HYP                                        STR00620
   14          VY(I,J)=JY(I,J)/HYP                                        STR00630
C                                                                         STR00640
CC                    SETMSG AND FACTOR PLOT SIZE                         STR00650
C                                                                         STR00660
          CALL SETMSG(2)                                                  STR00670
          CALL PLOTS(0,0,0)                                               STR00680
          CALL FACTOR(UNIT)                                               STR00690
C                                                                         STR00700
CC                    DRAW AXIS                                           STR00710
C                                                                         STR00720
          XOR=5.0                                                         STR00730
          YOR=10.0                                                        STR00740
       CALL NEWPEN(+4)                                                    STR00750
       CALL PLOT(XOR,YOR,+3)                                              STR00760
       X1=XOR+12.                                                         STR00770
       CALL PLOT(X1,YOR,+2)                                               STR00780
       Y1=YOR+6.                                                          STR00790
       Y2=YOR-6.                                                          STR00800
       CALL PLOT(XOR,Y1,+3)                                               STR00810
       CALL PLOT(XOR,Y2,+2)                                               STR00820
       CALL PLOT(XOR,YOR,+3)                                              STR00830
C                                                                         STR00840
CC                    COMPUTE NODE POSITIONS                              STR00850
C                                                                         STR00860
          DO 20 I=1,NC                                                    STR00870
   20     X(I)=FLOAT(I-1)                                                 STR00880
          DO 22 J=1,NS                                                    STR00890
   22     Y(J)=FLOAT(J-1)                                                 STR00900
C                                                                         STR00910
CC                    NEWPEN SETS LINE HEAVINESS (THICKNESS)              STR00920
C                                                                         STR00930
          CALL NEWPEN(+2)                                                 STR00940
C                                                                         STR00950
CC                    CALCULATE AND PLOT STREAM FUNCTION                  STR00960
```

```
C                                                                       STR00970
CC              + STEP ... FORWARD PROGRESSION (WITH DERIVATIVES)       STR00980
CC              - STEP ... BACKWARD PROGRESSION (AGAINST DERIVATIVES)   STR00990
C                                                                       STR01000
      TOLER=.03                                                         STR01010
      STEP=+.10                                                         STR01020
      NEWT=NS-2                                                         STR01030
      DO 500 JINK=1,NEWT                                                STR01040
      XLOC=XS(JINK)                                                     STR01050
      YLOC=YS(JINK)                                                     STR01060
      XPOS=XOR+XLOC                                                     STR01070
      YPOS=YOR+YLOC                                                     STR01080
      CALL PLOT(XPOS,YPOS,+3)                                           STR01090
      IJK=0                                                             STR01100
  400 CONTINUE                                                          STR01110
      IJK=IJK+1                                                         STR01120
      XP(IJK)=XLOC                                                      STR01130
      YP(IJK)=YLOC                                                      STR01140
      IF(JINK.EQ.1) WRITE(6,807) IJK,XLOC,YLOC                          STR01150
C                                                                       STR01160
CC              TEST FOR BOUNDARY                                       STR01170
C                                                                       STR01180
      IF(XLOC.LT.X(1).OR.XLOC.GT.X(NC)) GO TO 450                       STR01190
      IF(YLOC.LT.Y(1).OR.YLOC.GT.Y(NS)) GO TO 450                       STR01200
      CALL MARCH(STEP,XLOC,YLOC,XNEW,YNEW,TOLER)                        STR01210
      XPOS=XOR+XNEW                                                     STR01220
      YPOS=YOR+YNEW                                                     STR01230
      CALL PLOT(XPOS,YPOS,+2)                                           STR01240
      XLOC=XNEW                                                         STR01250
      YLOC=YNEW                                                         STR01260
      GO TO 400                                                         STR01270
  450 CONTINUE                                                          STR01280
C                                                                       STR01290
CC              DRAW IMAGE STREAM LINE                                  STR01300
C                                                                       STR01310
      XPOS=XOR+XP(1)                                                    STR01320
      YPOS=YOR-YP(1)                                                    STR01330
      CALL PLOT(XPOS,YPOS,+3)                                           STR01340
      DO 460 I=1,IJK                                                    STR01350
      XPOS=XOR+XP(I)                                                    STR01360
      YPOS=YOR-YP(I)                                                    STR01370
      CALL PLOT(XPOS,YPOS,+2)                                           STR01380
  460 CONTINUE                                                          STR01390
  500 CONTINUE                                                          STR01400
      CALL PLOT (0.,0.,+999)                                            STR01410
  800 FORMAT(6(I2,1X),F5.2,12A4)                                        STR01420
  810 FORMAT(' ',6(I2,1X),F5.2,12A4,/)                                  STR01430
  801 FORMAT(20A4)                                                      STR01440
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  802 FORMAT(3X,7(1X,E10.4))                                              STR01450
  807 FORMAT('  POINT(',I3,') =',2(2X,E10.4))                            STR01460
  809 FORMAT(I3,2(2X,E10.4))                                              STR01470
      STOP                                                                STR01480
      END                                                                 STR01490
C                                                                         STR01500
C           ***************************************                       STR01510
C           *                                     *                       STR01520
C           *          SUBROUTINE MARCH           *                       STR01530
C           *                                     *                       STR01540
C           ***************************************                       STR01550
C                                                                         STR01560
C            STEP.GT.0   FORWARD PROGRESSION OF STREAM FUNCTION           STR01570
C            STEP.LT.0   BACKWARD     "        "      "                   STR01580
C            XLOC,YLOC   PRESENT FIELD LOCATION                           STR01590
C            XNEW,YNEW   NEW LOCATION, STEPPED MAGNITUDE = STEP           STR01600
C            TOLER       REPRESENTS A DISTANCE USED TO DETERMINE          STR01610
C                         VECTOR INFLUENCE POSITIONS                      STR01620
C                          (SEE STMNTS USING TOLER)                       STR01630
C                                                                         STR01640
C                                                                         STR01650
C                                                                         STR01660
C                                                                         STR01670
C       INFLUENCE REGIONS CONVENTION                                      STR01680
C                                                                         STRC1690
C       380          340          370                                     STR01700
C           +.....................+                                       STR01710
C           .                     .                                       STR01720
C           .                     .                                       STR01730
C       310 .        300          . 330                                   STR01740
C           .                     .                                       STR01750
C           .                     .                                       STR01760
C           .                     .                                       STR01770
C           +.....................+                                       STR01780
C       350          320          360                                     STR01790
C                                                                         STR01800
C                                                                         STR01810
C                                                                         STR01820
C                                                                         STR01830
C                                                                         STR01840
      SUBROUTINE MARCH(STEP,XLOC,YLOC,XNEW,YNEW,TOLER)                    STR01850
      LOGICAL LOGIC(4)                                                    STR01860
      COMMON /SET/ VX(25,7),VY(25,7),X(25),Y(7)                           STR01870
      LOGIC(1)=.FALSE.                                                    STR01880
      LOGIC(2)=.FALSE.                                                    STR01890
      LOGIC(3)=.FALSE.                                                    STR01900
      LOGIC(4)=.FALSE.                                                    STR01910
      NS=7                                                                STR01920
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```
      NC=25                                                                  STR01930
      IM=AINT(XLOC)+1                                                        STR01940
      JM=AINT(YLOC)+1                                                        STR01950
      IF(ABS(X(IM)-XLOC).LT.TOLER) LOGIC(1)=.TRUE.                           STR01960
      IF(ABS(X(IM+1)-XLOC).LT.TOLER) LOGIC(3)=.TRUE.                         STR01970
      IF(ABS(Y(JM)-YLOC).LT.TOLER) LOGIC(2)=.TRUE.                           STR01980
      IF(ABS(Y(JM+1)-YLOC).LT.TOLER) LOGIC(4)=.TRUE.                         STR01990
      IF(LOGIC(1).AND.LOGIC(2)) GO TO 350                                    STR02000
      IF(LOGIC(2).AND.LOGIC(3)) GO TO 360                                    STR02010
      IF(LOGIC(3).AND.LOGIC(4)) GO TO 370                                    STR02020
      IF(LOGIC(4).AND.LOGIC(1)) GO TO 380                                    STR02030
C                                                                            STR02040
      IP=IM+1                                                                STR02050
      JP=JM+1                                                                STR02060
      IF(LOGIC(1)) GO TO 310                                                 STR02070
      IF(LOGIC(2)) GO TO 320                                                 STR02080
      IF(LOGIC(3)) GO TO 330                                                 STR02090
      IF(LOGIC(4)) GO TO 340                                                 STR02100
C                                                                            STR02110
  300 CONTINUE                                                               STR02120
CX    WRITE(6,730)                                                           STR02130
  730 FORMAT(' 300')                                                         STR02140
      IP=IM+1                                                                STR02150
      JP=JM+1                                                                STR02160
  303 D1=DIST(IM,JM,XLOC,YLOC)                                               STR02170
      D2=DIST(IP,JM,XLOC,YLOC)                                               STR02180
      D3=DIST(IP,JP,XLOC,YLOC)                                               STR02190
      D4=DIST(IM,JP,XLOC,YLOC)                                               STR02200
      DFX=VX(IM,JM)/D1+VX(IP,JM)/D2+VX(IP,JP)/D3+VX(IM,JP)/D4                STR02210
      DFY=VY(IM,JM)/D1+VY(IP,JM)/D2+VY(IP,JP)/D3+VY(IM,JP)/D4                STR02220
      GO TO 400                                                              STR02230
  310 IP=IM                                                                  STR02240
CX    WRITE(6,731)                                                           STR02250
  731 FORMAT(' 310')                                                         STR02260
      GO TO 303                                                              STR02270
  320 JP=JM                                                                  STR02280
CX    WRITE(6,732)                                                           STR02290
  732 FORMAT(' 320')                                                         STR02300
      GO TO 303                                                              STR02310
  330 IM=IP                                                                  STR02320
CX    WRITE(6,733)                                                           STR02330
  733 FORMAT(' 330')                                                         STR02340
      GO TO 303                                                              STR02350
  340 JM=JP                                                                  STR02360
CX    WRITE(6,734)                                                           STR02370
  734 FORMAT(' 340')                                                         STR02380
      GO TO 303                                                              STR02390
  350 DFX=VX(IM,JM)                                                          STR02400
```

```
CX      WRITE(6,735)                                                    STR02410
  735   FORMAT(' 350')                                                  STR02420
        DFY=VY(IM,JM)                                                   STR02430
        GO TO 400                                                       STR02440
  360   DFX=VX(IP,JM)                                                   STR02450
        DFY=VY(IP,JM)                                                   STR02460
CX      WRITE(6,736)                                                    STR02470
  736   FORMAT(' 360')                                                  STR02480
        GO TO 400                                                       STR02490
  370   DFX=VX(IP,JP)                                                   STR02500
        DFY=VY(IP,JP)                                                   STR02510
CX      WRITE(6,737)                                                    STR02520
  737   FORMAT(' 370')                                                  STR02530
        GO TO 400                                                       STR02540
  380   DFX=VX(IM,JP)                                                   STR02550
        DFY=VY(IM,JP)                                                   STR02560
CX      WRITE(6,738)                                                    STR02570
  738   FORMAT(' 380')                                                  STR02580
C                                                                       STR02590
C          NORMALIZE DIRECTIONAL DERIVATIVES                            STR02600
C                                                                       STR02610
  400   CONTINUE                                                        STR02620
        DER=(DFX*DFX+DFY*DFY)**.5                                       STR02630
        DFX=DFX/DER                                                     STR02640
        DFY=DFY/DER                                                     STR02650
CX      WRITE(6,700) IM,JM,IP,JP,DFX,DFY                                STR02660
  700   FORMAT(' IM=',I2,'  JM=',I2,'  IP=',I2,'  JP=',I2,              STR02670
     $    '    DFX= ',E10.4,'    DFY= ',E10.4)                          STR02680
        XNEW=XLOC+STEP*DFX                                              STR02690
        YNEW=YLOC+STEP*DFY                                              STR02700
        RETURN                                                          STR02710
        END                                                             STR02720
C                                                                       STR02730
CC            ****************************                              STR02740
CC            *                          *                              STR02750
CC            *      FUNCTION DIST       *                              STR02760
CC            *                          *                              STR02770
CC            ****************************                              STR02780
C                                                                       STR02790
        FUNCTION DIST(I,J,XL,YL)                                        STR02800
        COMMON /SET/ VX(25,7),VY(25,7),X(25),Y(7)                       STR02810
        D1=X(I)-XL                                                      STR02820
        D2=Y(J)-YL                                                      STR02830
        DIST=(D1*D1+D2*D2)**.5                                          STR02840
        RETURN                                                          STR02850
        END                                                             STR02860
```

```
01 10 03 10 01 01 00.50 01.00 F414A
 PHD 10.00       00.00       00.00
 NED 00.30       01.00       01.00
 NID 00.00       00.00       01.00
00 00 00 00 00 00 00.00 00.00 STOP
```

```
10 15  3 10  1 01  0.17  1.00 F4148
(11)          VOLTAGE POTENTIAL
 1 0.1000D+02 0.2100D+01 0.1482D+01 0.1141D+01 0.9210D+00 0.7686D+00 0.6633D+00
 2 0.3280D+01 0.1779D+01 0.1347D+01 0.1084D+01 0.8916D+00 0.7490D+00 0.6461D+00
 3 0.1409D+01 0.1330D+01 0.1163D+01 0.9873D+00 0.8339D+00 0.7088D+00 0.6113D+00
 4 0.1002D+01 0.1028D+01 0.9575D+00 0.8497D+00 0.7373D+00 0.6341D+00 0.5443D+00
 5 0.7464D+00 0.7889D+00 0.7651D+00 0.7027D+00 0.6240D+00 0.5413D+00 0.4598D+00
 6 0.5460D+00 0.5963D+00 0.5975D+00 0.5641D+00 0.5099D+00 0.4441D+00 0.3712D+00
 7 0.3911D+00 0.4434D+00 0.4575D+00 0.4418D+00 0.4046D+00 0.3522D+00 0.2879D+00
 8 0.2745D+00 0.3244D+00 0.3442D+00 0.3387D+00 0.3130D+00 0.2712D+00 0.2156D+00
 9 0.1888D+00 0.2336D+00 0.2546D+00 0.2546D+00 0.2366D+00 0.2033D+00 0.1563D+00
10 0.1273D+00 0.1656D+00 0.1853D+00 0.1880D+00 0.1753D+00 0.1489D+00 0.1101D+00
11 0.8428D-01 0.1156D+00 0.1328D+00 0.1364D+00 0.1273D+00 0.1067D+00 0.7548D-01
12 0.5474D-01 0.7953D-01 0.9376D-01 0.9738D-01 0.9085D-01 0.7491D-01 0.5043D-01
13 0.3489D-01 0.5395D-01 0.6525D-01 0.6846D-01 0.6375D-01 0.5163D-01 0.3287D-01
14 0.2183D-01 0.3611D-01 0.4478D-01 0.4742D-01 0.4403D-01 0.3498D-01 0.2092D-01
15 0.1341D-01 0.2386D-01 0.3033D-01 0.3239D-01 0.2997D-01 0.2333D-01 0.1302D-01
16 0.8091D-02 0.1557D-01 0.2029D-01 0.2183D-01 0.2012D-01 0.1533D-01 0.7934D-02
17 0.4803D-02 0.1005D-01 0.1341D-01 0.1454D-01 0.1333D-01 0.9947D-02 0.4743D-02
18 0.2808D-02 0.6423D-02 0.8766D-02 0.9563D-02 0.8732D-02 0.6378D-02 0.2787D-02
19 0.1619D-02 0.4063D-02 0.5667D-02 0.6219D-02 0.5652D-02 0.4045D-02 0.1613D-02
20 0.9215D-03 0.2543D-02 0.3619D-02 0.3993D-02 0.3613D-02 0.2536D-02 0.9197D-03
21 0.5166D-03 0.1569D-02 0.2273D-02 0.2520D-02 0.2271D-02 0.1566D-02 0.5162D-03
22 0.2830D-03 0.9413D-03 0.1386D-02 0.1542D-02 0.1385D-02 0.9402D-03 0.2829D-03
23 0.1466D-03 0.5277D-03 0.7866D-03 0.8780D-03 0.7862D-03 0.5273D-03 0.1466D-03
24 0.6207D-04 0.2362D-03 0.3549D-03 0.3969D-03 0.3548D-03 0.2360D-03 0.6209D-04
25 0.0        0.0        0.0        0.0        0.0        0.0        0.0
(11)          ELECTRON DENSITY
 1 0.1500D+00 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01
 2 0.5252D+00 0.6450D+00 0.8505D+00 0.9368D+00 0.9676D+00 0.9784D+00 0.9811D+00
 3 0.5804D+00 0.7537D+00 0.8860D+00 0.9487D+00 0.9747D+00 0.9847D+00 0.9873D+00
 4 0.8255D+00 0.8812D+00 0.9352D+00 0.9672D+00 0.9830D+00 0.9898D+00 0.9916D+00
 5 0.9332D+00 0.9479D+00 0.9674D+00 0.9815D+00 0.9897D+00 0.9936D+00 0.9947D+00
 6 0.9736D+00 0.9776D+00 0.9845D+00 0.9904D+00 0.9942D+00 0.9962D+00 0.9968D+00
 7 0.9891D+00 0.9904D+00 0.9928D+00 0.9952D+00 0.9969D+00 0.9978D+00 0.9981D+00
 8 0.9954D+00 0.9958D+00 0.9967D+00 0.9976D+00 0.9984D+00 0.9988D+00 0.9990D+00
 9 0.9980D+00 0.9982D+00 0.9985D+00 0.9989D+00 0.9992D+00 0.9994D+00 0.9994D+00
10 0.9991D+00 0.9992D+00 0.9993D+00 0.9995D+00 0.9996D+00 0.9997D+00 0.9997D+00
11 0.9996D+00 0.9996D+00 0.9997D+00 0.9997D+00 0.9998D+00 0.9998D+00 0.9998D+00
12 0.9996D+00 0.9998D+00 0.9999D+00 0.9999D+00 0.9999D+00 0.9999D+00 0.9999D+00
13 0.9999D+00 0.9999D+00 0.9999D+00 0.9999D+00 0.1000D+01 0.1000D+01 0.1000D+01
14 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01
15 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01
16 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01
17 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01
18 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01
19 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01
20 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01 0.1000D+01
```

```
21 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01
22 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01
23 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01
24 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01
25 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01
(11)          ION DENSITY
 1 0.0         0.0          0.0          0.0          0.0          0.0          0.0
 2 0.0         0.26810D+00  0.80250D+00  0.92440D+00  0.95910D+00  0.97070D+00  0.97380D+00
 3 0.12640D+00 0.73060D+00  0.89750D+00  0.95620D+00  0.97890D+00  0.98750D+00  0.98980D+00
 4 0.80840D+00 0.89420D+00  0.94510D+00  0.97270D+00  0.98610D+00  0.99180D+00  0.99330D+00
 5 0.94060D+00 0.95580D+00  0.97280D+00  0.98480D+00  0.99160D+00  0.99480D+00  0.99580D+00
 6 0.97770D+00 0.98140D+00  0.98720D+00  0.99210D+00  0.99530D+00  0.99690D+00  0.99740D+00
 7 0.99100D+00 0.99210D+00  0.99410D+00  0.99610D+00  0.99750D+00  0.99830D+00  0.99850D+00
 8 0.99630D+00 0.99660D+00  0.99730D+00  0.99810D+00  0.99870D+00  0.99910D+00  0.99920D+00
 9 0.99840D+00 0.99850D+00  0.99880D+00  0.99910D+00  0.99930D+00  0.99950D+00  0.99960D+00
10 0.99930D+00 0.99930D+00  0.99940D+00  0.99960D+00  0.99970D+00  0.99970D+00  0.99980D+00
11 0.99970D+00 0.99970D+00  0.99970D+00  0.99980D+00  0.99980D+00  0.99990D+00  0.99990D+00
12 0.99990D+00 0.99990D+00  0.99990D+00  0.99990D+00  0.99990D+00  0.99990D+00  0.99990D+00
13 0.99990D+00 0.99990D+00  0.99990D+00  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
14 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
15 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
16 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
17 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
18 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
19 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
20 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
21 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
22 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
23 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
24 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
25 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
(11)          TEMP DIST (TRUNCATED)
 1 0.12360D+01 0.10930D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
 2 0.10950D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
 3 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
 4 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
 5 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
 6 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
 7 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
 8 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
 9 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
10 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
11 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
12 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
13 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
14 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
15 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
16 0.10000D+01 0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01  0.10000D+01
```

```
 17 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01
 18 0.10000D+01 0.10C0D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01
 19 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01
 20 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01
 21 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01
 22 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01
 23 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01
 24 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01
 25 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01 0.10000D+01
00
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